Modeling Cosmic Bubble Collisions

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Abstract

Particle physics uses field theories to describe matter. Different values of a scalar field, for example, can be associated with different phases. A bubble in a field is an area inside which the field is different from its value in the surrounding space. After bubbles form, they expand and collide according to the scalar field equation of motion, but the process of expansion and the effects of their collisions are impossible to predict analytically. We wrote a modified version of LatticeEasy, a program that evolves scalar fields on a lattice, to model two types of early-universe phase transitions. In the first, the potential is a function of a single field. As bubbles in this field expand and collide, gravitational radiation is produced. Our program outputs the resulting gravitational wave spectra. In other simulations, the value of one scalar field represents the radius of extra dimensions; if the value is small, the extra dimensions are compact, as theorized by particle physics. The potential function has two minima at roughly equal finite values of the field, and also reaches another minimum as the field value approaches infinity, where the dimensions are large. Decompactification will occur if, when the bubble walls collide, the field in the collision area is kicked over the domain wall and grows to infinity.

Scalar Fields and LatticeEasy

A field assigns a property to every point in space. One example is the electric field, which exerts a force on charged particles. It is a vector field, because the force has both magnitude and direction, but the values of a field can also be scalars. Temperature, for example, could be represented as a scalar field, since a temperature measurement is simply a number with no associated direction. Cold water will warm when it is put in a hot room; likewise, the value of a field at a point is affected by the values at surrounding points. When potential energy is a function of field value, the value at a point will also change if it is not a minimum of the potential function: if potential energy would be lower at a different field value, the field will shift into that nearby minimum. A field will be stable and never change when three conditions are present: the field value is the same at every point (the field is homogeneous), the value is at the absolute minimum of the potential function, and the field is not moving. More interesting cases occur when the field values are not at a potential minimum, or if some points are in a different minimum from surrounding points. The evolution of the field is described by the Klein-Gordon equation:

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} - \frac{\nabla^2\phi}{a^2} + \frac{\partial V}{\partial \phi} = 0$$

LatticeEasy is a C++ program that uses this equation to evolve scalar fields on a lattice. In our simulations, the field is initialized by assigning a value to each point in a cube of 256 or 512 points on a side. I added new output functions to save information such as the field value and energy density at regularly spaced points in the lattice. Mathematica is used to produce 3D plots and contour plots of these data to illustrate the evolution of the field.

References & Acknowledgements

[1] A. Kosowsky, M. S. Turner and R. Watkins, Phys. Rev. D **45**, 4514 (1992).
[2] S. Coleman and F. De Luccia, Phys. Rev. D **21**, 3305 (1980).
[3] A. Aguirre, M. C. Johnson and M. Larfors, Phys. Rev. D **81**, 043527 (2010) [arXiv:0911.4342 [hep-th]]

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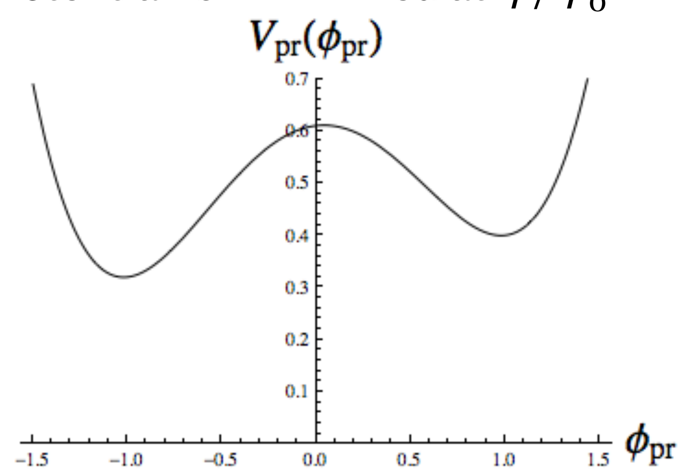
Gravitational Radiation

Grand unified theory predicts that the weak and strong nuclear forces and the electromagnetic force all act as one at very high energies. The energy scale of the early Universe exceeded the grand unification energy. But as the Universe cooled, the strong nuclear force became distinct from the others through a phase transition.

We model the spectrum of gravity waves produced during the phase transition by assuming a potential of the form

$$V(\phi) = \frac{\lambda}{8}(\phi^2 - \phi_0^2)^2 + \varepsilon\lambda\phi_0^3(\phi + \phi_0) + \alpha\lambda\phi_0^4$$

Potential is minimized at $\phi/\phi_o \approx \pm 1$:



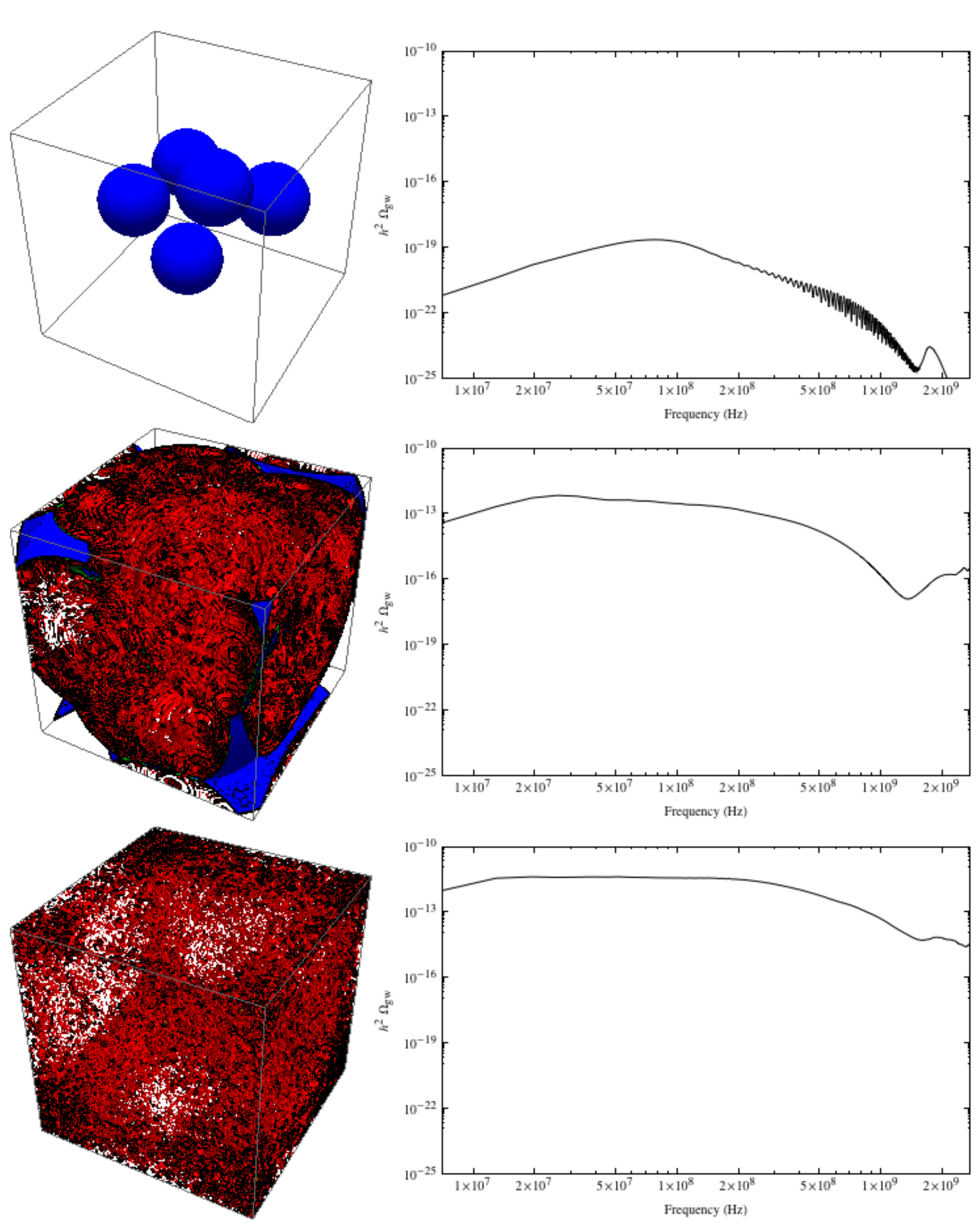
The forces are unified everywhere in the early Universe, so all space is in the higher, local minimum ($\phi/\phi_o \approx 1$). Quantum physics allows some space to tunnel through the domain wall to the lower minimum, creating bubbles of true minimum with initial radius [1]

$$R_0 = \frac{1}{\sqrt{\lambda}\varepsilon\phi_0}$$

The initial bubble profile is given by [2]

$$\phi(r) = \phi_0 \tanh\left[\frac{1}{2}\phi_0\sqrt{\lambda}(r - R_0)\right]$$

The parameters α, ε, and ϕ_o are chosen so that the field energy is a fraction of the energy scale, which is set to 10^{12} GeV, 10^{13} GeV, or 10^{14} GeV by adjusting λ. LatticeEasy is programmed to generate random positions for the centers of five bubbles. After the bubbles are initialized, the field is evolved for one Hubble time (long enough for light to travel from one side of the box to the other). Below are the gravitational wave spectrum and field contours at the beginning, middle, and end of a simulation at 10^{14} GeV:



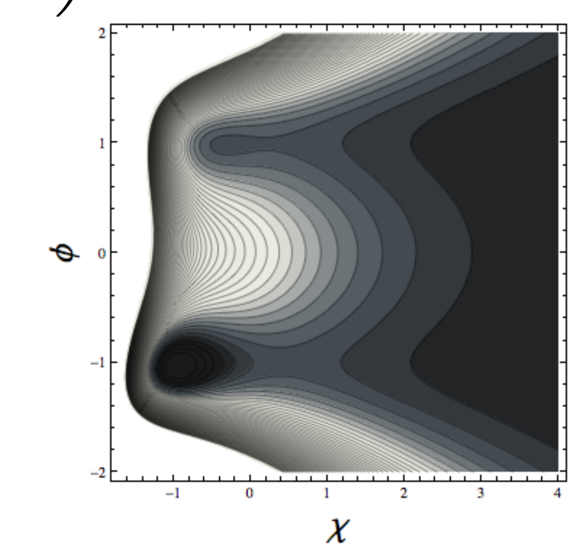
Decompactification

A human tightrope walker can only move along the rope in one dimension, walking back and forth. But an ant is small enough to crawl around on the surface of the rope, moving in two dimensions. Like this second dimension, whose radius is too small for the human to experience it, there may be extra spatial dimensions in our Universe. One of the three we do experience may also once have been small. Decompactification, the process by which small dimensions become large, may occur during bubble collisions.

A toy potential

$$V(\phi,\chi) = e^{-n\chi/M_\chi}\mu_\phi^4\left(\frac{\phi^2}{M_\phi^2} - 1\right)^2 + \mu_\chi^4\left[-e^{-2\chi/M_\chi} + ae^{-\chi/M_\chi} + be^{-3\chi/M_\chi}\right] + \delta\mu_\chi^4 e^{-3\chi/M_\chi}\left(\frac{\phi}{M_\phi} + A\right)$$

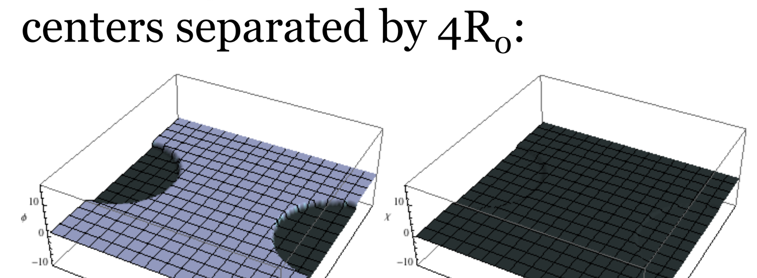
has two minima at finite values of ϕ and χ, and a third minimum at infinitely large values of χ:



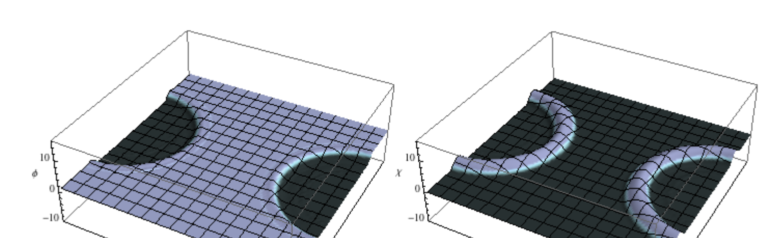
The value of the χ field represents the radius of extra dimensions. All space is initially in one of the two minima at finite values of χ, so the extra dimensions are small. If colliding bubble walls supply enough kinetic energy to kick the χ field over the domain wall to infinity, the extra dimensions decompactify in the collision region.

In one simulation:

① Two bubbles are initialized with radius R_o (given in [3]) and their centers separated by $4R_o$:

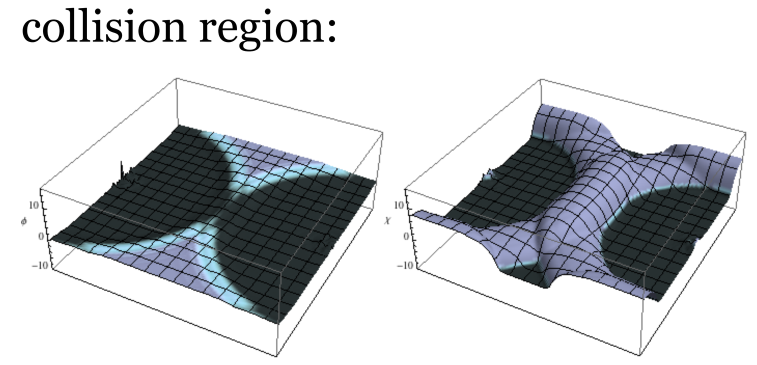


② As the bubbles begin to expand, the bubble wall relaxes to the actual instanton solution:

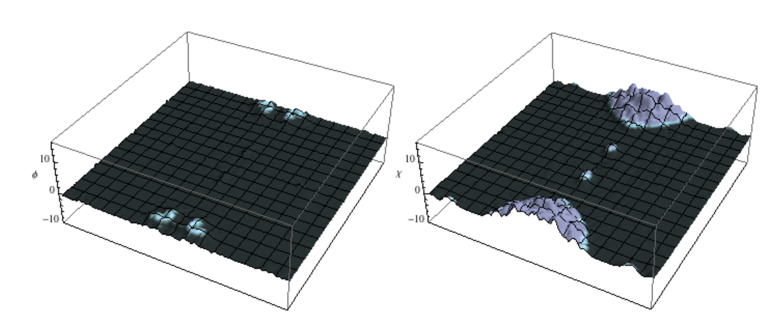


A ridge appears in the χ field at the edge of the bubble because the shortest path between the two minima is not a straight line along a constant value of χ. The actual bubble profile involves an increase in the χ value, a path that is difficult to predict analytically but can be seen early in the simulation.

③ When the bubbles first collide, the χ field is raised up in the collision region:



④ But after the collision, the χ field collapses back to a finite value:



If the bubbles nucleate farther apart, the kinetic energy of the bubble walls is greater when they collide. When the centers are separated by $5R_o$, the χ field still collapses back after the collision. The simulations so far have not resulted in decompactification.