Electroweak Symmetry Breaking and Lattice Simulations

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Abstract

The universe has undergone several phase transitions during its evolution. One, called the electroweak phase transition, occurs at an energy scale of 100 GeV when the age of the universe is 10^{-12} s. During this phase transition the fermions that make up the Universe are given mass. We are able to model this phase transition as a first order phase transition by evolving the Higgs field, a scalar field that mediates this phase transition. In order to explore this phase transition we have coupled the incompressible non-relativistic Navier-Stokes equations to the Higgs field. These fluid equations model the particles that are present in the universe at the time of the phase transition. We have written a program that evolves the Higgs field and the Navier-Stokes equations on a 3-dimensional lattice using a Runge-Kutta 2nd order method of integration.

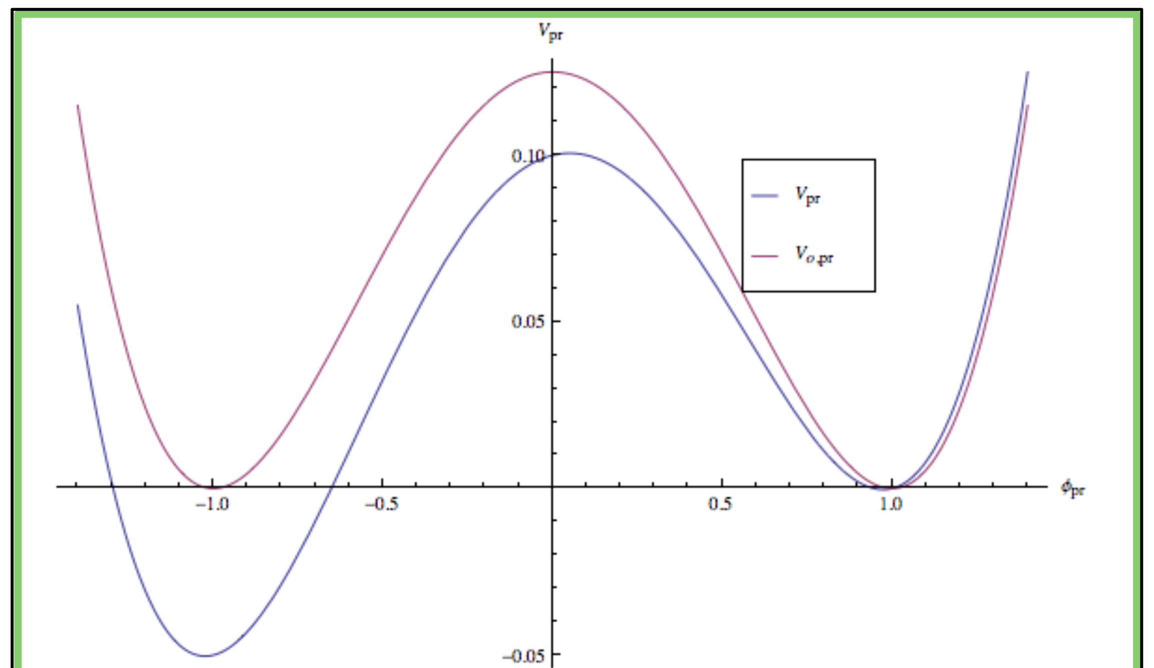

Figure 1. Quartic (Perturbed) Potential

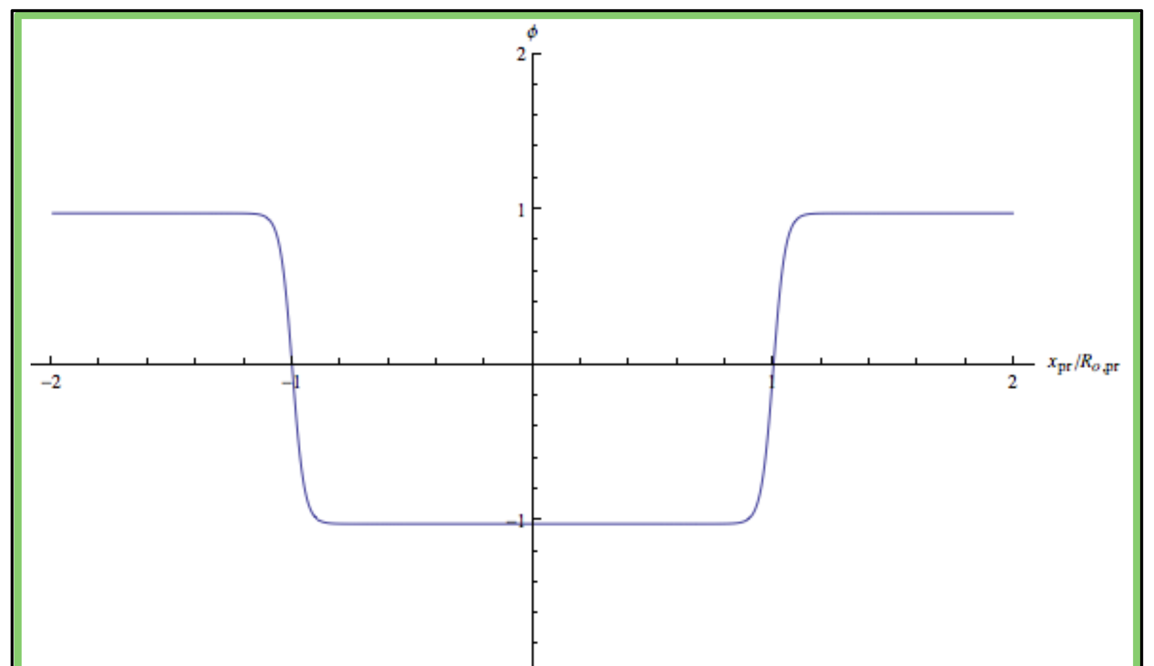

Figure 2. 1-D Slice of Initial Field

Symmetry Breaking and Phase Transitions

All known interactions between particles are governed by the four fundamental forces: electro-magnetic, weak, strong, and gravitational. As we look back to the early universe, there is a time at which these four forces are indistinguishable from each other. As the universe ages, the temperature lowers and energy dilutes, these forces began to decouple. First the gravitational force becomes independent, then the strong force, and finally the weak and electromagnetic forces decouple. When the weak force brakes from the electromagnetic, all of the fermions (electrons, quarks, leptons, etc) in the standard model acquire mass. This process is mediated by the Higgs Field (Boson) undergoing a first order phase transition. We can evolve a field that represents the Higgs boson. This field, in a simplified model, will have stable equilibria corresponding to different stable phases of the system. We call the higher minimum the false vacuum state (where the electromagnetic and weak forces are unified), and the lower minimum the true vacuum state. During this time the Higgs field tunnels from the false vacuum to the true vacuum.

Results

Using the lattice evolution C++ program we wrote for this problem we have done many simulations at 128^3 and 256^3 resolutions at varying values for the kinematic viscosity (η) and the coupling constant (ζ) for our model. We present here a sampling of our data. Figure 3 shows a large sample of the bubble radii as a function of time for different ζ and η. We see at lower ζ the bubble walls reach greater velocities (the slope) and are generally not dependent on η. However at higher ζ as seen in Figure 4 we see very interesting trends. Our bubble walls are actually slowing down and at the very end of our runs the velocities, just barely, turn negative. In other words the bubbles start to collapse in on themselves due to a large interaction with the fluid. Figure 5 shows a slice at a program time of 200 for the field profile the velocity modulus and vector plot and the fluid density with $\zeta = \eta = 0.1$. The fluid density plot shows us that the density of the fluid inside the bubble is just slightly less than the density of the fluid outside the bubble.

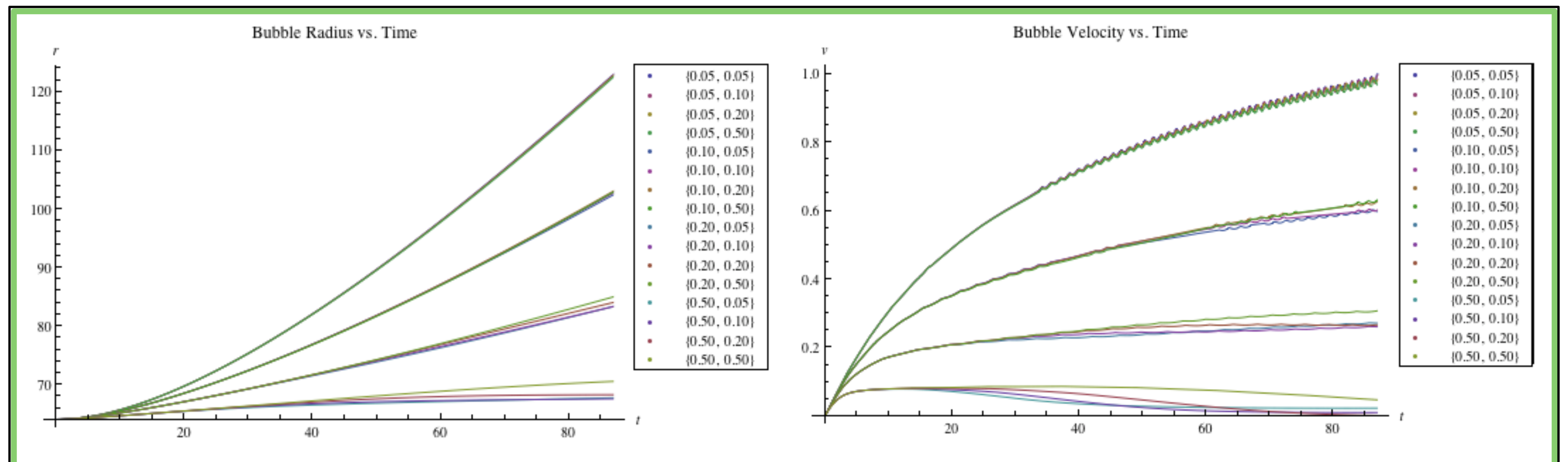

Figure 3. Sample of Bubble Radius and Velocity vs. Time at 256^3 Resolution

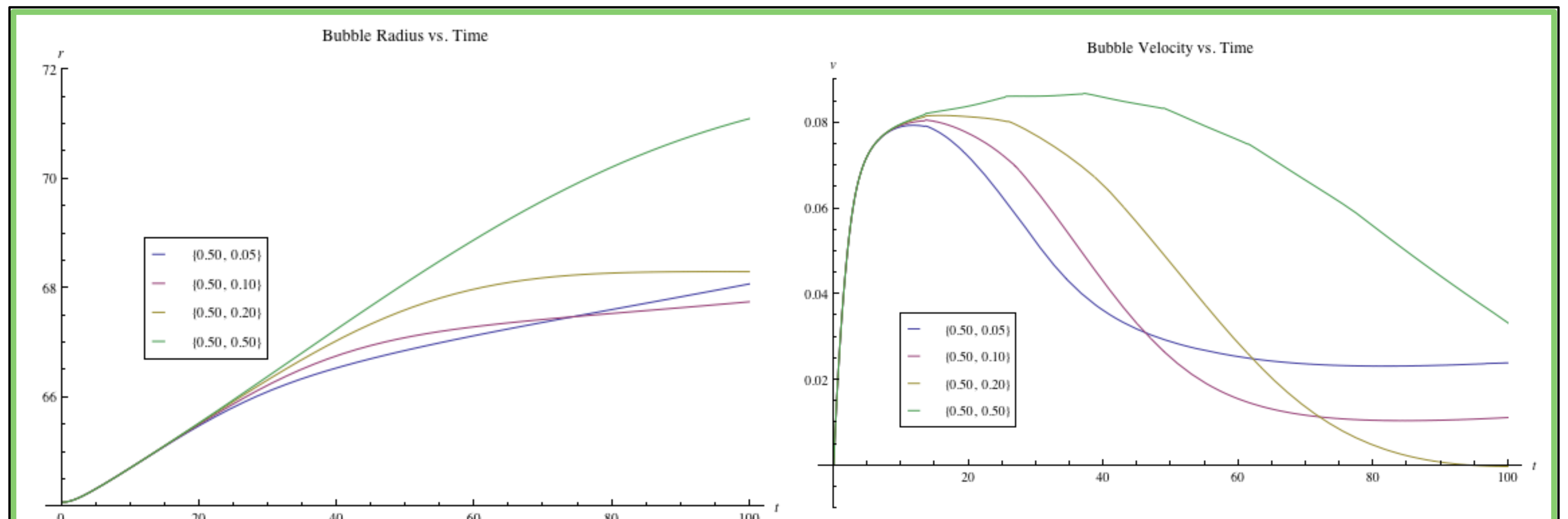

Figure 4. Bubble Radius and Velocity for ζ=0.5 and $\eta \in \{0.05, 0.1, 0.2, 0.5\}$

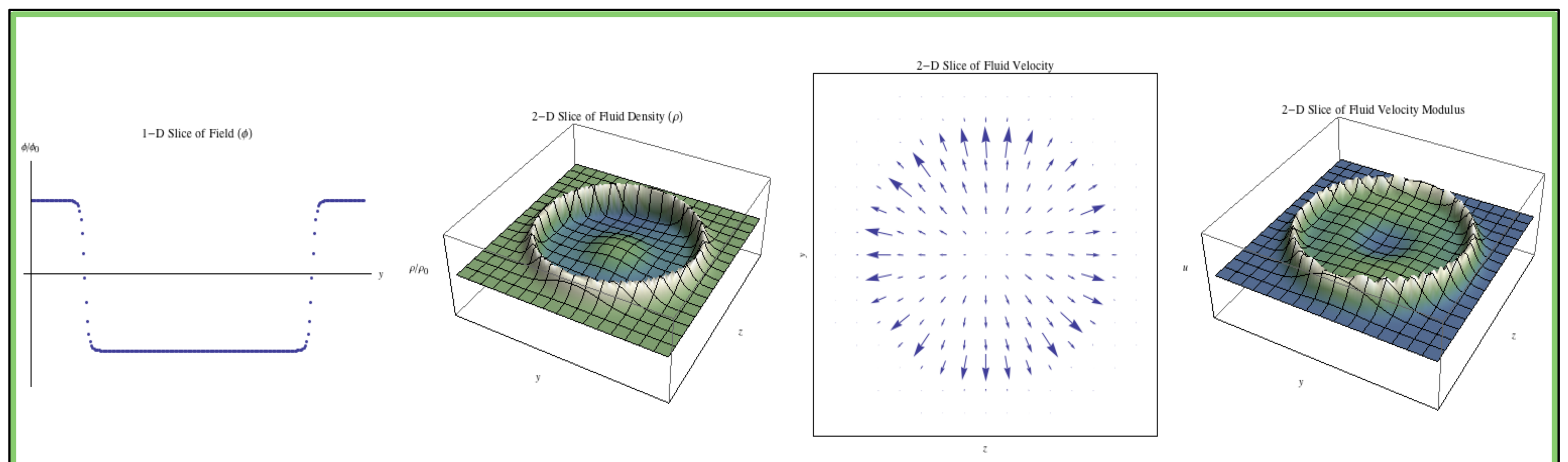

Figure 5. Slice of Run with $\zeta = \eta = 0.1$ at $t_{pr} = 200$ and a Resolution of 256^3

Where To Now?

We see from our results that interesting physics is happening at $\zeta \approx 0.5$. The bubble walls are actually slowing down and acquiring negative velocities, and thus the surrounding space is staying at a higher potential. Exploring the parameter space around this value is a logical next step for this project. We are also planning to use a more intricate potential than our perturbed quartic well.

The Model

In our particular model we are not evolving the Higgs Field in isolation but have coupled it to a fluid to see the effects of interactions with other particles in the universe. To model this interaction we will treat the particles in the universe as a fluid. In order to simulate the phase transition we need: a potential energy function for the field, the Klein–Gordon equation with a term that couples to the fluid, the Navier-Stokes equation with a term that couples to the field, and the continuity equation to evolve the fluid energy density. Note that in the simulations we did not include expansion, for the time scale of expansion was many orders of magnitude greater than the time scale of the simulations. Throughout this project we used a toy model for the potential given by a quadratic potential with a linear perturbation [1]:

$$V(\phi) = \frac{\lambda}{8}\left(\phi^2 - \phi_0^2\right)^2 + \epsilon\lambda(\phi - \phi_0)\phi_0^3$$

Note that λ and ε are model dependent parameters and ϕ is our field so ϕ_0 is the initial field value. The Klein-Gordon equation with the fluid coupling term (underlined), is used to evolve our field [1,2,3,5]:

$$\ddot{\phi} = \nabla^2\phi - \frac{dV}{d\phi} - \zeta \underline{u \cdot \nabla\phi}$$

Note that ζ is the fluid-field coupling constant and u is the fluid velocity. A full relativistic treatment of the Navier-Stokes equation is very difficult to evolve without making certain assumptions about the symmetry of the problem. To avoid unnecessary symmetry assumptions we used the Navier-Stokes equations for a non-relativistic irrotational incompressible fluid, with no viscous force. The added coupling term is underlined [3,5]:

$$\frac{\partial u_i}{\partial t} = \frac{1}{\rho}\left(\eta\nabla^2 u_i - \nabla P - \underline{\zeta(u\cdot\nabla\phi)\frac{\partial\phi}{\partial i}}\right)$$

Note that η is the kinematic viscosity of the fluid and the pressure P is given by the equation of state $P=w\rho$ where w = 1/3 and ρ is the energy density of the fluid. Using a non-relativistic Navier Stokes equation is a fine assumption because the maximum velocity of the fluid (around 0.2c) is within the non-relativistic regime. To evolve the energy density we just used the continuity equation:

$$\frac{\partial\rho}{\partial t} = -\nabla\cdot(\rho u)$$

The bubble was nucleated with a profile according the following equation at the start of the simulation (Figure 2) [1]:

$$\phi = \phi_0 \tanh\left(\frac{1}{2}\phi_0\sqrt{\lambda}(r - R_0)\right)$$

Here r is the spherical coordinate of the point and R_0 is the initial bubble radius.

References and Acknowledgements

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