Abstract

Lattice simulations are very useful computational cosmology, because they let us test theories and solve differential equations that may not have analytic solutions. We have a very good lattice simulation code: GABE (the Grid And Bubble Evolver), which lets us test cosmological theories on an expanding background. One drawback to this code is the assumption of a flat space-time, which observation of the universe (and our existence) tells us is not realistic. In response to this, we modified GABE to include Newtonian perturbations to the space-time metric (gravitational perturbations), and tested several theories with this new code.

The Perturbed FLRW Metric

In cosmology, the choice of a spacetime metric is crucial to the outcome of our simulations and experiments. The most common choice is known as the Freidmann-Lemaitre-Robertson-Walker Metric, which is given by

\[
g_{\mu\nu} = \begin{pmatrix}
-1 & 0 & 0 & 0 \\
0 & a^2 & 0 & 0 \\
0 & 0 & a^2 & 0 \\
0 & 0 & 0 & a^2
\end{pmatrix}
\]

(1)

This metric depends only on time, meaning that it is uniform over all of space. However, evidence (such as gravitational lensing, the existence of galaxies, and our own existence) tells us that in reality, there is a spatial dependence to the metric.

To account for this, we can modify the FLRW Metric to include spatial components.

\[
g_{\mu\nu} = \begin{pmatrix}
-(1 + 2\Psi) & 0 & 0 & 0 \\
0 & a^2(1 - 2\Psi) & 0 & 0 \\
0 & 0 & a^2(1 - 2\Psi) & 0 \\
0 & 0 & 0 & a^2(1 - 2\Psi)
\end{pmatrix}
\]

(2)

Where \(\Psi\) and \(\Phi\) are known as the Bardeen variables. Both are dependent on both space and time, so this adds a spatial component to our metric. \(\Psi\) is the Newtonian gravitational potential at a point, and \(\Phi\) is equal to zero when there is no anisotropic stress present.

Scalar Fields and Lattice Simulations

In computational cosmology, we often find it useful to view the universe as a collection of fields, because they are much easier to simulate than particles. We have a program called GABE: the Grid And Bubble Evolver, which evolves the equations of motion of scalar fields with the second order Runge-Kutta method. We calculate the value of the fields at specific points, in a three dimensional grid. This is known as a Lattice Simulation.

This solves the horizon problem, but introduces another: at the end of inflation, the universe has nothing familiar to us in it: no matter, no electromagnetic waves.

Future Work

Now that we have a fully working code, we are working towards more new and exciting applications for our code. We have talked to researchers at MIT and Tufts about using our code to test theories involving scalar field dark matter as a method for the formation of structure in the early universe.

Acknowledgements and References

I would like to thank the Kenyon Summer Science Scholars Program, my lab mates, Eva, Zach, and Furqan, and Professor Tom Giblin for working with me on this project.