*Psi*zing up the Universe: The Effects of the the Gravitational Potential after Inflation

Caroline Popiel '18 and Professor Tom Giblin
Department of Physics, Kenyon College, Gambier, OH

Abstract

Early in its history, the universe grew by dozens of *e*-foldings in just a fraction of a second, an event called inflation. Somewhere down the line, the universe evolved to its present state and humans started looking up at the stars with their telescopes. But do we know anything else that happened between this early stage of the universe and now?

After inflation came a period of time called reheating. The inflaton field, the field that drove inflation, expanded so quickly that we need a way to account for the high temperature of the universe after inflation.

During my summer research, I examined the process of reheating, specifically by taking the effect of local gravity, $\boldsymbol{\psi}$, into consideration. By calculating the local gravitational potential at each point in a simulation of the early universe, it is possible to perturb the metric that describes the expansion of the universe. Accounting for small perturbations allows a more accurate description of the evolution of the universe.

In my project, I detail the differences between the Friedmann-Lemaître-Robertson-Walker metric (the 'usual' way of describing expansion) and the linear perturbed metric that accounts for gravity. I analyze the results of both simulations in order to see what effect these linear perturbations have during reheating.

FLRW Universe

The Friedmann-Lemaître-Robertson-Walker (FLRW) universe is characterized by being flat, homogeneous, and isotropic. The expansion rate is the same at every point in the universe. The expansion can be described by the FLRW metric

$$ds^2 = -dt^2 + a^2(t)[dx^2 + dy^2 + dz^2],$$

Where t is the time component, x, y, and z are the spacial components, and a is the scale factor. This metric yields the Klein-Gordon equation of motion

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} - \frac{\nabla^2}{a^2}\phi = -\frac{dV}{d\phi},$$

where ϕ is the inflaton field. This equation of motion can be modeled by Kenyon's program GABE (Grid and Bubble Evolver), which is a C++ program that evolves scalar fields on an expanding background. It simulates the evolution of the universe for this model and for others.

Perturbed Universe

As opposed to the FLRW universe, consider a perturbed universe. This universe is curved and inhomogeneous, and it takes the Newtonian gravitational potential, $\boldsymbol{\psi}$, into consideration as an even better approximation for an expanding universe. Because of this, expansion is *not* the same everywhere, as it is influenced by local gravity. So, the perturbed metric is

$$ds^2 = -(1+2\psi)dt^2 + a^2(t)(1-2\varphi)[dx^2 + dy^2 + dz^2],$$

and we will assume no anisotropic stress, which means $\varphi = \psi$. The linearized perturbed equation of motion that corresponds to this metric is

$$\ddot{\phi} = -\frac{dV}{d\phi}(2\psi + 1) + \frac{\nabla^2}{a^2}\phi(4\psi + 1) - 3H\dot{\phi} + 4\dot{\phi}\dot{\psi}$$

where $H = \frac{\dot{a}}{a}$. We can see that if $\psi = 0$, we will recover the Klein-Gordon equation of motion.

The potential that is used in GABE for both the FLRW and perturbed universes is

$$V(\phi, \chi) = \frac{1}{2}m^2\phi^2 + \frac{1}{2}g^2\phi^2\chi^2,$$

where χ is a generic scalar field. This quadratic potential is a simple toy model. It features a coupling term, g, which allows interaction between the ϕ and χ fields.

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Newtonian Gravitational Potential

The Newtonian gravitational potential, $\boldsymbol{\psi}$, is a function of both space and time, which is why expansion is not the same everywhere. $\boldsymbol{\psi}$ is strongly influenced by the energy density of the universe, ρ:

$$\nabla^2\psi = 4\pi G a^2 \delta\rho.$$

We assume that ψ stays small (hence or ability to use it in perturbation theory), so only linear-order terms are considered.

The differential equations for ψ are

$$2\frac{\nabla^2\psi}{a^2} - 3H\dot{\psi} = \frac{8\pi G}{3}\delta\rho \text{ and}$$

$$\dot{\psi} = 4\pi G\dot{\bar{\phi}}\delta\phi - H\psi.$$

ψ can then be solved for in GABE using a series of Fourier transformations.

One way to see the effect that ψ has on the simulation is by allowing backreaction. With no backreaction, ψ is calculated, but the EOM is not perturbed, so the value of ψ has an influence on the evolution of the simulation. With backreaction, ψ is calculated and we're using the perturbed EOM, so because the perturbed EOM depends on ψ, ψ influences the evolution of the simulation. Figure 1 shows the variance, maximum, and minimum values for ψ, and shows the effect that backreaction has.

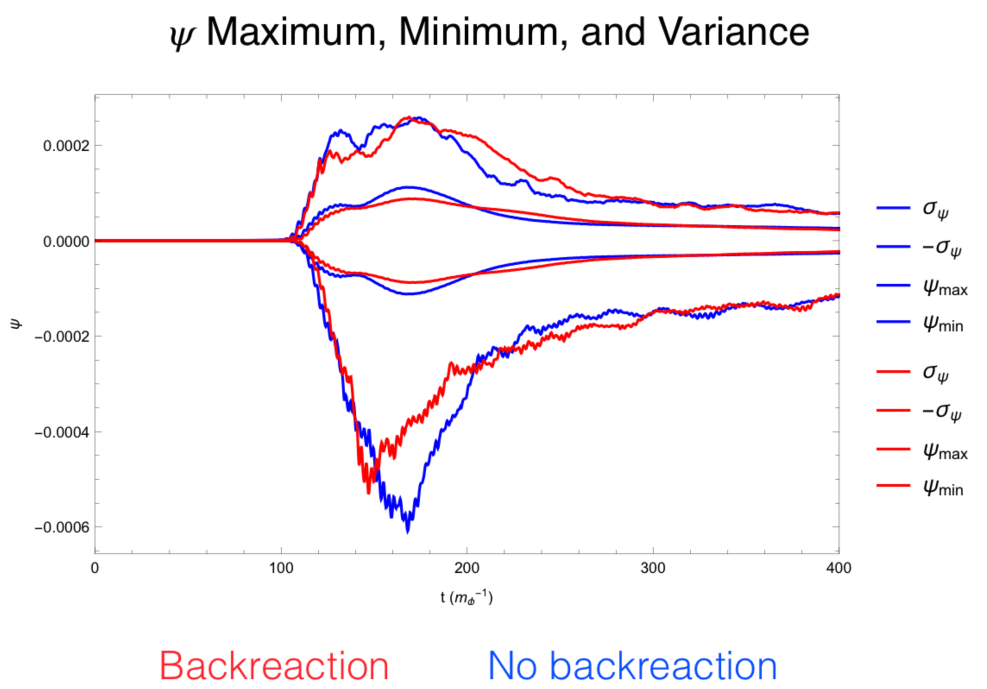

Figure 1 Values for the gravitational potential, ψ

In addition, we can also examine the relationship between ψ and the energy density ρ. Figure 2 demonstrates that high energy density corresponds to low values of ψ, and low energy density corresponds to higher values of ψ. In both Figure 1 and Figure 2, ψ never gets to be more than $\sim|10^{-4}|$, which means that perturbations stay small and should not have a large effect on the simulation of reheating.

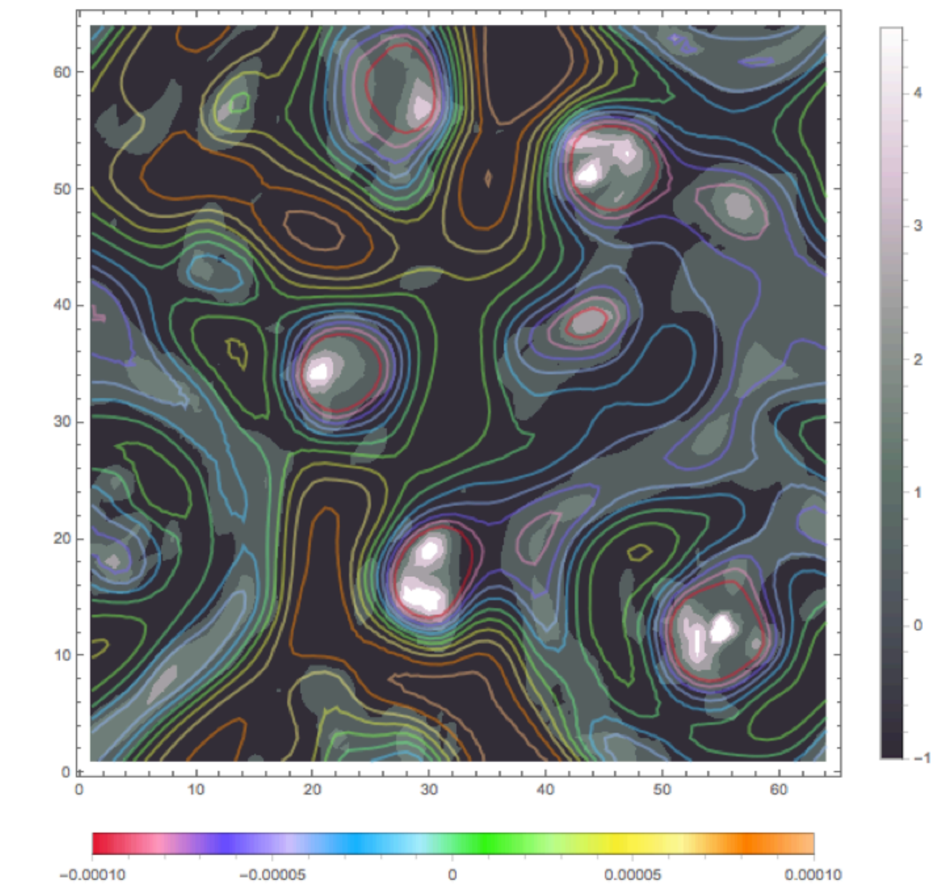

Figure 2 The color bar on the right shows the fractional energy density of the universe, $\frac{\rho - \bar{\rho}}{\bar{\rho}}$, where darker colors correspond to underdensities and lighter colors correspond to overdensities. The color bar on the bottom shows ψ, where green is ~ 0.

References

- D. Baumann, "Cosmology; Ch 4: Cosmological Perturbation Theory," http://www.damtp.cam.ac.uk/user/db275/Cosmology.pdf
- R. Easter, R. Flauger, J. Gilmore, "Delayed Reheating and the Breakdown of Coherent Oscillations," arXiv[1003.3011v2]
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Reheating With ψ

Compared to an FLWR run, the perturbed EOM shows only small variations (see Figure 3), which can be expected due to the small values that ψ has.

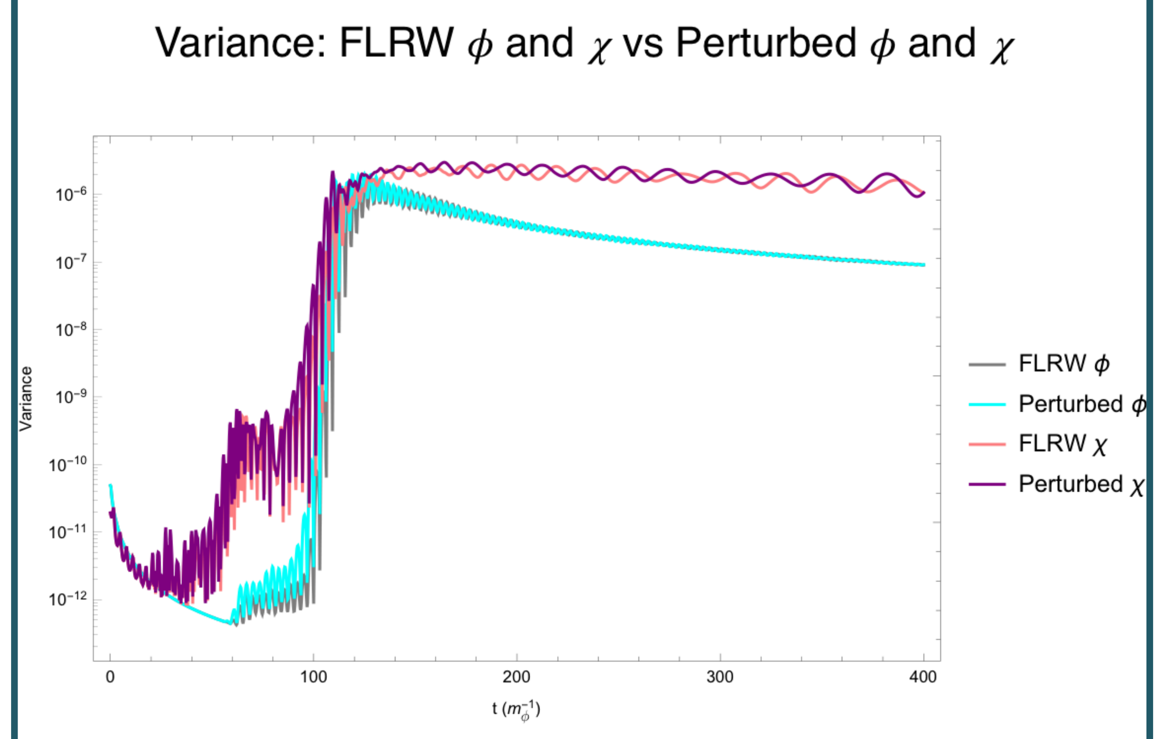

Figure 3 Reheating run for FLRW and perturbed EOMs, at a box size of $L = 4{m_\phi}^{-1}$

When the box size is increased, modes with longer wavelengths are included in the initial field spectra (see Figure 4), which makes the simulation quickly become unphysical.

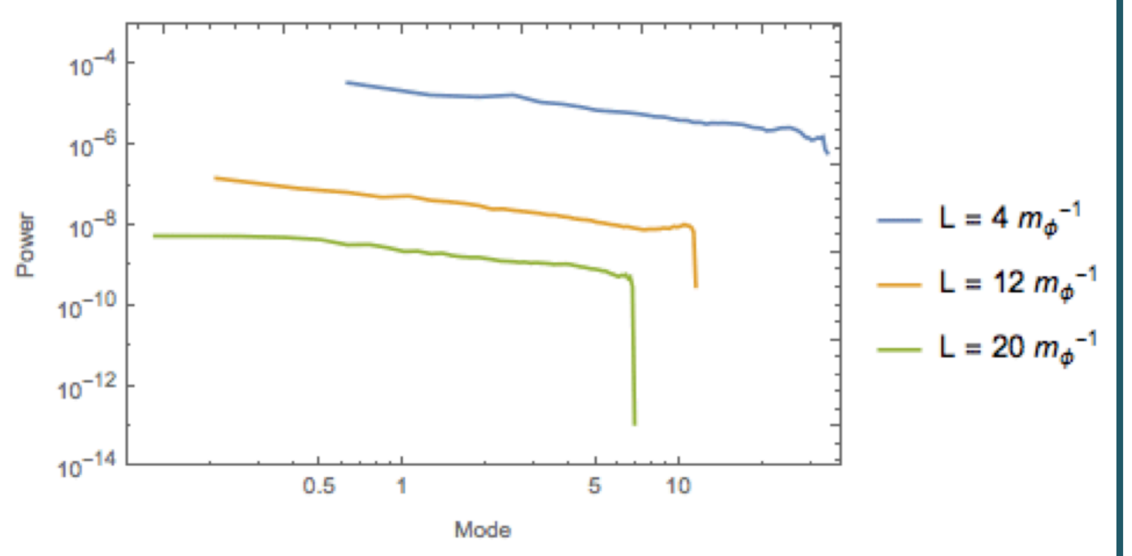

Figure 4 Power for various box sizes

By changing the effective mass, it is possible to modify the initial field spectra, but it is not enough save the simulation from physical behavior.

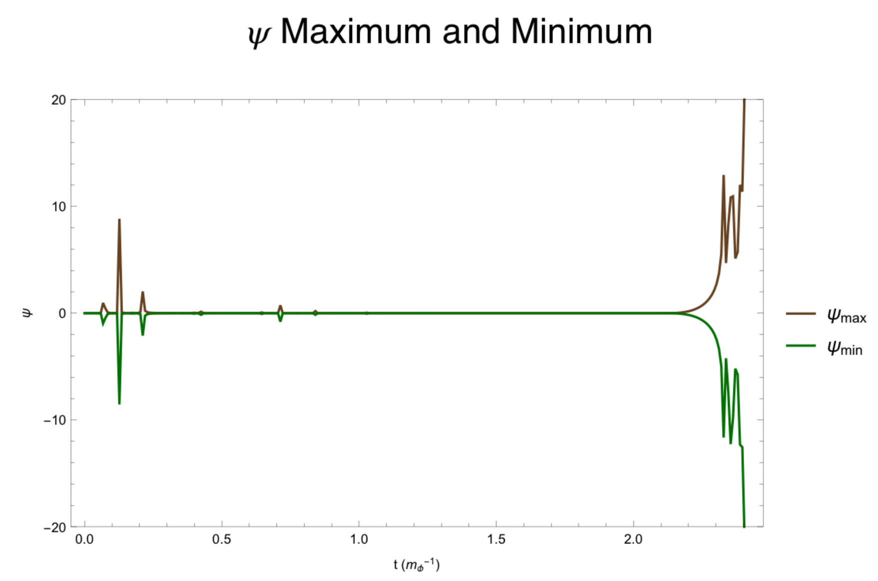

Figure 5 At a box size of $L = 120{m_\phi}^{-1}$, the simulation lasts for less than $3{m_\phi}^{-1}$ before $\psi > 1$ (compared to being about to run for $400{m_\phi}^{-1}$ or longer). When $\psi > 1$, or even gets close to 1, perturbation theory breaks down and the simulation no longer exhibits physical behavior, even for the modified effective mass.

Conclusions

The gravitational potential does indeed have varying effects on the evolution of the early universe, the extent of which depend on box size and other parameters. To be able to fully examine and expand upon the ideas presented here, it will be necessary to go beyond the linearized perturbed EOM to include non-linear terms, and to find a more accurate way to calculate ψ and $\dot{\psi}$. In addition, full general relativity will probably have to be taken into consideration.