Population growth models

<table>
<thead>
<tr>
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<th>Unrestrained population growth</th>
<th>Limited population growth</th>
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</thead>
<tbody>
<tr>
<td>Geometric growth</td>
<td>$N_t = N_0 \lambda^t$</td>
<td>$N_t = N_0 e^{r_{\text{max}}t}$</td>
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<tr>
<td>Logistic growth</td>
<td>$N_{t+1} = N_t \left[1 + r_{\text{max}} \left(1 - \frac{N}{K}\right)\right]$</td>
<td>$\frac{dN}{dt} = r_{\text{max}} N \left(1 - \frac{N}{K}\right)$</td>
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Note: Diskert logistic is not in the book!

What is $t$ in the discrete generation models?
What is $t$ in the overlapping generations models?

Under what conditions does unlimited vs. limited growth apply?
Geometric and exponential growth not generally tenable in the long term...
What are the implications for human population growth

**Logistic growth**

What sets the carrying capacity $K$?
How is this related to Damuth’s rule for how observed population density changes with body size?
How might this interact further with how $r_{\text{max}}$ changes with body size?
The logistic equation, as a differential equation, smoothly approaches an equilibrium population size of $K$. But many natural populations show some degree of fluctuation even around that equilibrium.

May be due to:

1. Interactions with competitors, predators, or prey populations (later in the semester…)
2. Environmental fluctuations…”stochasticity”
3. Chaos…

So what is chaos? Specifically *Deterministic Chaos*

Even simple, completely deterministic mathematical models of population dynamics, under certain conditions, can exhibit patterns that are for all intents and purposes *unpredictable*.


Discovered that the discrete logistic (and other discrete population models) display deterministic chaotic behavior.

The behavior of the model is VERY sensitive to the value of $r_{\text{max}}$.

- **Low intrinsic rate of increase (< 2):** much like continuous logistic, some dampening oscillations around $K$.

- **Intrinsic rate between 2 and about 2.8:** “period doubling” from persistent 2 point cyclical fluctuations to 4, 8, and 16 point cycles.

- **Intrinsic rate above 2.8:** Chaotic fluctuations that *appear random*. Important fact: in the chaotic regime, very slight differences in the initial population (which we can usually not know exactly) will lead to very different patterns of fluctuations. This is termed *sensitivity to initial conditions*.

But does chaos occur in nature? How can we tell it from fluctuations driven by the other factors?

Where would you look for it (in what sorts of organisms and under what sorts of circumstances?)

Why is this important… what are the limits of the knowable and the predictable in science.

**Time permitting: More on Human populations**