Reminders:
HW due on Friday, posted online
Read 5.1 for Wednesday (skip 4.5 unless you just have to know!)

Exercise – Probability Distributions of Discrete and Continuous Random Variables

Means and Variances for Discrete Random Variables

Mean – “Expected Value” – of Discrete Random Variables
• If X is a DRV taking on values \( x_1, x_2, x_3, \ldots, x_k \) with probabilities \( p_1, p_2, p_3, \ldots, p_k \)
• The mean (or Expected Value) of X is
  \[
  \mu = E[X] = x_1*p_1 + x_2*p_2 + x_3*p_3 + \ldots + x_k*p_k.
  \]

Law of Averages (or the Law of Large Numbers) – The sample mean calculated from many independent trials will converge on (get closer to) the mean of the probability distribution, \( E[X] \), as the sample size \( n \) gets larger. The more accurate the desired estimate of the mean, the larger the number of samples necessary.

Application of Expectation and the law of Averages – Lotteries and Casinos
• Example: Keno
  o 100 numbered balls (1,2,3,\ldots,99,100) in a hopper
  o Randomly select 25 balls
  o Bet $1 on a number and the payoff is $3 if the number is in the 25 drawn at random
  o What is the mean payoff?

Variance of Discrete Random Variables
• If X is a DRV taking on values \( x_1, x_2, x_3, \ldots, x_k \) with probabilities \( p_1, p_2, p_3, \ldots, p_k \) and mean (as described above) \( \mu_X \).
• The variance of X is
  \[
  \sigma^2 = Var[X] = (x_1-\mu_X)^2*p_1+(x_2-\mu_X)^2*p_2+(x_3-\mu_X)^2*p_3+\ldots+(x_k-\mu_X)^2*p_k
  \]
  • Reminder: the standard deviation of X is the square root of the variance.
  \[
  \sigma = SD[X] = \sqrt{(x_1-\mu_X)^2*p_1+(x_2-\mu_X)^2*p_2+(x_3-\mu_X)^2*p_3+\ldots+(x_k-\mu_X)^2*p_k}\]

Linear Transformation and Random Variables
If X is a random variable and a and b are constants…
• \( E[a+bX] = a+bE[X] \) is the mean of the transformed variable
• \( Var[a+bX] = b^2Var[X] \) is the variance of the transformed variable
• \( SD[a+bX] = bSD[X] \) is the standard deviation of the transformed variable
So linear transforms with random variables are the same as with descriptive statistics in Ch. 1…
Combining Random Variables

If $X$ and $Y$ are random variables...
- $E[X+Y] = E[X] + E[Y]$

If $X$ and $Y$ are independent random variables...
- $Var[X+Y] = Var[X] + Var[Y]$ and
- $Var[X-Y] = Var[X] + Var[Y]$
  THEY ARE THE SAME

If $X$ and $Y$ are random variables with correlation $\rho$ (rho)
- $Var[X+Y] = Var[X] + Var[Y] + 2\rho SD[X]*SD[Y]$ and
- $Var[X-Y] = Var[X] + Var[Y] – 2\rho SD[X]*SD[Y]$
  THEY ARE NOT THE SAME

Return to Keno application:
Suppose you bet on two separate games of Keno simultaneously. Find the mean and variance and standard deviation of the payoff.

Exercise: In Monopoly, Sorry, Parchisi, Backgammon, and many other games, you roll two dice to determine how far you move each turn. Find the mean, variance, and standard deviation of the number of spaces you will move in your first six turns.