Reminders:
HW due on Friday, posted online
HW hint – minitab gives you access to dice with any number of sides
   Calc > Random Data >Integers select from 1 to N for N-sided
Reading for Friday: 5.2

Final points of Random Variables

Law of Averages (or the Law of Large Numbers) – The sample mean calculated from many independent trials will converge on (get closer to) the mean of the probability distribution, E[X], as the sample size n gets larger. The more accurate the desired estimate of the mean, the larger the number of samples necessary.

Linear Transformation and Random Variables
   If X is a random variable and a and b are constants…
      • $E[a+bX] = a+bE[X]$ is the mean of the transformed variable
      • $\text{Var}[a+bX] = b^2\text{Var}[X]$ is the variance of the transformed variable
      • $\text{SD}[a+bX] = b\text{SD}[X]$ is the standard deviation of the transformed variable
So linear transforms with random variables are the same as with descriptive statistics in Ch. 1…

Sampling Distributions: Any statistic from a random sample or experiment is a Random Variable. The distribution of the values of that statistic is its sampling distribution.

Sampling Distributions for Counts

Bernoulli trials – random binary trial, outcome is always one of two possibilities (eg.: yes or no, 0 or 1, present or absent, male or female, alive or dead, success or failure)

Sample space of a Bernoulli trial?

Binomial experiments – repeated Bernoulli trials
   1. There are n trials (observations)
   2. The n trials are independent
   3. The probability of “success” (p) is constant

Binomial distribution – If X is the number of successes in n independent Bernoulli trials, then $X=B(n,p)$ and the probability of k successes is defined by the binomial distribution

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$
where
\[
\binom{n}{k} = \frac{n!}{k!(n-k)!}
\] is the binomial coefficient “n choose k”

and \( n! = n \times (n-1) \times (n-2) \times \ldots \times 2 \times 1 \) is the factorial

Luckily, you can use minitab to do binomial calculations …
Calc > Probability Distributions > Binomial

Why is this important?: The sampling distribution of a count is Binomial!!

If a population contains a proportion \( p \) of a particular outcome (a “success”), the count \( X \) of that outcome in a sample of size \( n \) is approximately described by the binomial distribution \( B(n, p) \). The approximation gets better as the size of the population grows (relative to the sample).

**Binomial mean and standard deviation for sample counts.**

For \( X=B(n, p) \)

\[
\mu_X = np \\
\sigma_X = \sqrt{np(1-p)}
\]

is the mean of \( X \)

is the standard deviation of \( X \)

**Sample Proportions**

- \( \text{counts / sample size } X/n=B(n,p)/n \)

**Mean and standard deviation of sample proportions**

Consider a constant sample size \( n \) and use the linear transformation rules for the means and variances of random variables. The proportion is \( Y=a+bX \)

where \( a=0 \) and \( b=1/n \), i.e., \( Y=X/n \).

\[
\mu_Y = \frac{\mu_X}{n} = \frac{np}{n} = p
\]

is the mean of the sample proportion

\[
\sigma_Y = \frac{\sigma_X}{n} = \frac{\sqrt{np(1-p)}}{n} = \sqrt{\frac{p(1-p)}{n}}
\]

is its standard deviation

**Handout: Binomial practice problems**

**You are the Dean of Admissions:** Suppose that Kenyon college would like to have a first-year entering class of 415 students next year. Past experience shows that about 27% of the students admitted will accept Kenyon’s offer. Answer the following questions assuming that the college decides to admit 1,400 students and the students make their decisions independently.

1. What is the distribution of the number of students who accept?
2. What are the mean and standard deviation of the number of students who accept?
3. What is the probability that the College gets more students than it wants?
4. Given the number of students admitted, what is the standard deviation for the proportion of students who will accept?