Exercise: Just a coincidence? Where is the weirdness threshold?

1. Psychic Probability (EESEE exercise 5.68 on P. 378)
Researchers in parapsychology must be both wary of tricksters and able to discount instances of luck when investigating evidence of psychic ability. This is especially true in the testing of individuals who claim to possess these extraordinary abilities. In one such test of psychic ability, Don, McDonough, and Warren (1992) claim to have eliminated the possibility of trickery by a self-proclaimed psychic and report his ability exceeded that dictated by chance.

In selection tasks predicting which of four equally probable shapes (star, wave, cross, or circle) had been randomly chosen by a computer, the subject (a self proclaimed psychic) was told when he was correct (a hit). His scores in 12 sessions of 24 “cards” over three days are as follows:

<table>
<thead>
<tr>
<th>All trials</th>
<th>Trials</th>
<th>Hits</th>
<th>Prop</th>
<th>Expected</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>288</td>
<td>88</td>
<td>0.3056</td>
<td></td>
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</tbody>
</table>

Questions:
1. Assuming independent trials and a subject that is simply guessing, what would you expect the proportion to be if the shapes all have equal probability of being selected?
2. How many hits would the psychic have to get to convince you? This is the same as asking how improbable an outcome would have to be considered different from our expectation based on a random process. Consider this in terms of cumulative probability and find your threshold for being “convinced” by this psychic.
   a. What is your probability model? ______________
   b. What is your probability threshold? ______________
   c. How many would he have to get right? __________

2. Birthday Coincidences – NOTE: This Binomial approximation is NOT actually appropriate since the trials are NOT independent! Still it is instructive.

When people get together, how often will two people at the gathering share a birthday? Ignoring leap-year babies, consider the case of equal probabilities of birth for each day of the year, 1/365 = 0.0027397.

1. Given the 28 people in the room, there are 378 possible pairs of people (found using the binomial coefficient!). What is the chance that two of us share a birthday?
   a. How many pairs of people are in the room? __________
   b. For each pair, what is the probability that their birthdays are the same? (Hint: Use the complement. What is the probability that a pair do not share a birthday?) __________
   c. What is our probability model? ______________
d. What is the probability that at least 1 pair share a birthday? ____________

2. What is your threshold for a “freaky” coincidence here? ______________
   a. How many pairs of shared birthdays would it take to FREAK YOU OUT? ______________

3. The Loaded Dice
Back when we started talking about probability theory, a few groups had questions about whether the dice they were rolling were fair.
   1. What is the probability model for 100 rolls of a fair die landing on 6? ____________
   2. How many sixes would it take in a hundred roles to convince you that a die was not fair? ______________ Based on what probability level? ______________.