5 /5 1. Wolverine’s challenge: “We mutants will grow and eventually replace you insignificant Normal human beings. You have nothing to say about it, so relax—Mutants produce only one child every other year, so in a hundred years you’ll see only fifty of us.”

What do you say to Wolverine? [Many answers were accepted.]

Wolverine’s math sounds like an underestimate. Suppose a Mutant is reproductively active for 20 years, a family size of 10 (a conservative estimate; others are possible). Suppose children start reproducing after 20 years. A hundred years is then approximately 5 generations (other ideas possible).

Using the equation $N_n = N_0 Q^n$ where $N_0$ is the initial population, $Q$ is the number of offspring produced per generation, and $n$ is the number of generations, we can see that there will instead be $10^5$ mutants in a hundred years.

$N_n = N_0 Q^n$

$N_n = (1)(10)^5$

$N_n = 10^5$

[The answer should be consistent with one’s assumptions, and quite large.]

[Note: This very rough calculation doesn’t include the need for two parents. The family size would then be defined as 20 children from two parents.]

In a perfect environment such as the Enterprise, Tribbles do not age, so they live forever after producing each litter (unless they eat poisoned grain.) Therefore, after each “generation” there are ten plus one = eleven tribbles.

15 /15 2. Uhura brought one tribble onto the Enterprise.

(a) Each tribble produced a litter of 10 (and kept living) every 12 hours. Calculate how many tribbles there should be in 5 days.

$N_n = N_0 Q^n$

$N_n = (1)(11)^{10}$

$N_n = 2.59 \times 10^{10}$
There should be \(2.59 \times 10^{10}\) tribbles aboard the Enterprise in 5 days.

(b) Set up an Excel file to calculate the number of tribbles produced over 5 days. In Excel, plot your results (number of tribbles over time). Print a copy of the plot and the data table.

Excel sheet is attached.

(c) What actually became of the tribble population on the Enterprise, and why?

Eventually the tribble population died out on the Enterprise because: They ate poisoned grain; a “virus” killed them (actually not the same as poison; this contradiction is inherent in the film); they eventually they ran out of food, so they all starved to death. (Any of these are good answers.)

[Note: Technically this happened on the space station, not the Enterprise. If anyone knows the whole episode, they may object that actually the ones on the Enterprise were beamed onto the Klingon ship, but that’s ok.]

5 /5 3. During the poisoning, the population of tribbles drops to 100. Assume those tribbles are still producing 10 offspring per litter, but only half survive the poison, and the parent dies after one litter. What is their new doubling time?

If only 5 survive every 12 hours, and the parent dies:

Using the equation for doubling time, \(N_t = N_0 2^{t/d}\), with \(N_t\) being population size after time \(t\), \(N_0\) being the initial population size, \(t\) being the time between generations, and \(d\) being the time for doubling, we can see that the new doubling time for the tribbles would be 5.17 hours.

\[N_t = N_0 2^{t/d} = N_0 Q^n\]
\[2^{t/d} = Q^n\]
\[2^{12\text{hr/dhr}} = 5^1\]
\[\log_{10}(2^{12\text{hr/dhr}}) = \log_{10}(5)\]
\[(12\text{hr/dhr})\log_{10}(2) = \log_{10}(5)\]
\[d\text{ hr} = (12\text{hr})\log_{10}(2)/\log_{10}(5)\]
\[d\text{ hr} = 5.17\text{ hours}\]

10 /10 4. In an environment like the Enterprise:

a) How many tribbles would you need initially to produce 1 billion tribbles in 2 days?
\[ N_n = N_0 Q^n \]

1,000,000,000 = \( N_0 (11^4) \)

\( N_0 = \frac{10^9}{14641} \)

\( N_0 = 68,300 \)

Therefore, there would need to be 68,300 tribbles initially to have at least 1,000,000,000 tribbles in 2 days.

b) How many hours would it take for 1 tribble to produce one billion tribbles?

\[ 1,000,000,000 = (1)(11^n) \]

\[ n \log(11) = \log(1,000,000,000) \]

\[ n = \frac{\log(1,000,000,000)}{\log(11)} \]

\[ n = 8.64 \]

It would take 8.64 generations, or 104 hours, for 1 tribble to produce one billion tribbles.

5 /5 5. In real-life natural environments, what phenomena limit the size of growing populations? State three different examples.

In real-life environments, there are many phenomena that keep populations under control.

- Every organism has natural predators, which help control the population of the organism.
- Diseases that can spread and limit the size of the organism’s population.
- As in the case of the tribbles on the Enterprise, there are a limited number of resources in an environment, and these resources can only maintain a certain amount of organisms in one environment, limiting the size of the population.
- When a population starts to run up against limits, individuals begin to disperse. The original location then holds a smaller population than exponential growth predicts (although the overall number of individuals everywhere is still growing.)

10 /10 6. Tribbles are hermaphrodites (can fertilize themselves, or another tribble). They come in different coat colors, determined genetically. Suppose that
Black color is dominant to White, but Brown is codominant with White (a brown allele plus a white allele makes a cream-colored tribble).

(a) If a pure black tribble mates with a white tribble, what proportion of offspring will be black?

[Note: Other definition of symbols is ok.]

BB x ww → Bw only

Because the black color is dominant to the white, all of the offspring from these two tribbles will be black-coated (though carriers of white).

(b) If a black offspring from cross (a) fertilizes itself, what proportion of offspring will most likely be black?

Bw x Bw → BB, Bw, wB, ww

The offspring from (a) will have one black allele and one white allele, meaning that there is only a 25% chance that the offspring of itself will be white-coated, leaving the other 75% to most likely be black-coated (25% with two black alleles and 50% with one black and one white allele).

(c) If a cream-colored tribble fertilizes itself, what colors of offspring will appear, in what approximate proportions?

CC = brown, Cw = cream

Cw x Cw → CC, Cw, wC, ww

A cream-colored tribble has one brown allele and one white allele, meaning that if it fertilizes itself, about 25% of its offspring will have two brown alleles, making it brown, about 50% will have one brown and one white allele, making it cream-colored, and about 25% will have two white alleles, making it white-coated.