The Cosmological Origin of Primordial Magnetic Fields

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Abstract

Telescopes have revealed the existence of very weak magnetic fields in the intergalactic medium. However, current astrophysical knowledge does not call for such phenomena: the intergalactic medium is charge-neutral and thus sourceless, and planetary and celestial magnetic fields vanish at very short distances. So the origin of these fields remains an open question. We investigate one possible explanation: that they may have been created at the end of the Inflationary Epoch. Early in the Universe's history, it underwent a period of rapid exponential expansion called inflation. Many inflationary models predict a post-inflationary phenomenon called preheating. In preheating, energy from the scalar inflaton field is transferred into other fields as it experiences sinusoidal decay. The consequences of preheating have proven quite successful in enhancing the viability of inflationary models. So it seems plausible that preheating could have created primordial magnetic fields. To test this hypothesis, we wrote a lattice simulation that evolves the equations of motion for the inflaton, a massless scalar field, and the electromagnetic 4-potential.

Scalar Fields and Inflationary Dynamics

A scalar field is a function that associates a scalar value with every point in a given spacetime. The classic example is a temperature field defined across a room: every part of the room has some temperature. We can use scalar fields to analyze many interesting phenomena in cosmology. One such example is the theory of inflation. It states that, sometime between 10^{-33} and 10^{-31} seconds after the Big Bang, the Universe experienced extremely rapid expansion, increasing in volume by a factor of around 10^{78}. Inflationary theory provides compelling solutions to many problems in contemporary cosmology, making it a widely accepted hypothesis. We can model inflation by filling the Universe with a single scalar field, called the inflaton. As an inhomogeneous fundamental degree of freedom, the inflaton obeys the Klein-Gordon equation:

$$\Box^2\phi + \frac{\partial V}{\partial \phi} = 0$$

We add to our model a second fundamental degree of freedom. The two fields have a potential of the form:

$$V(\phi,\chi) = \frac{1}{2}m^2\phi^2 + \frac{1}{2}g^2\phi^2\chi^2$$

We use the Friedmann equations to calculate the growth of the scale factor. Note that, for the time frames that we care about, the most convenient units of the inflaton are Planck masses.

Field Initialization and Perturbations

During inflation, the inflaton and other fields are approximately homogeneous. After inflation, this state of homogeneity ceases. It can be shown that the time-derivative of the inflaton vanishes at a homogeneous value of $\phi \approx .2m_{pl}$. By starting our simulation around this time, we ensure that we need not worry about large initial time-derivatives. In addition, inflation ceases at $\phi \approx .16m_{pl}$. So the phenomena we wish to model will appear shortly after we begin. We will also need to induce random perturbations in our fields. We do this in the following manner: For each field in our model, we generate two waves that move across the momentum space of our model Universe, in opposite directions. Each point on these waves deviates from its expected value by a small, random value. We transform the sum of these two fields into small inhomogeneities.

References and Acknowledgments

[1] arXiv:1005.5322v4
[2] arXiv:1006.3504
[3] arXiv:0911.4771
[4] arXiv:0907.0197v4
[5.] arXiv:0712.2991v1
[6.] arXiv:hep-ph/9910410v1
[7] http://www.psc.edu/science/Glatzmaier/field_big.gif

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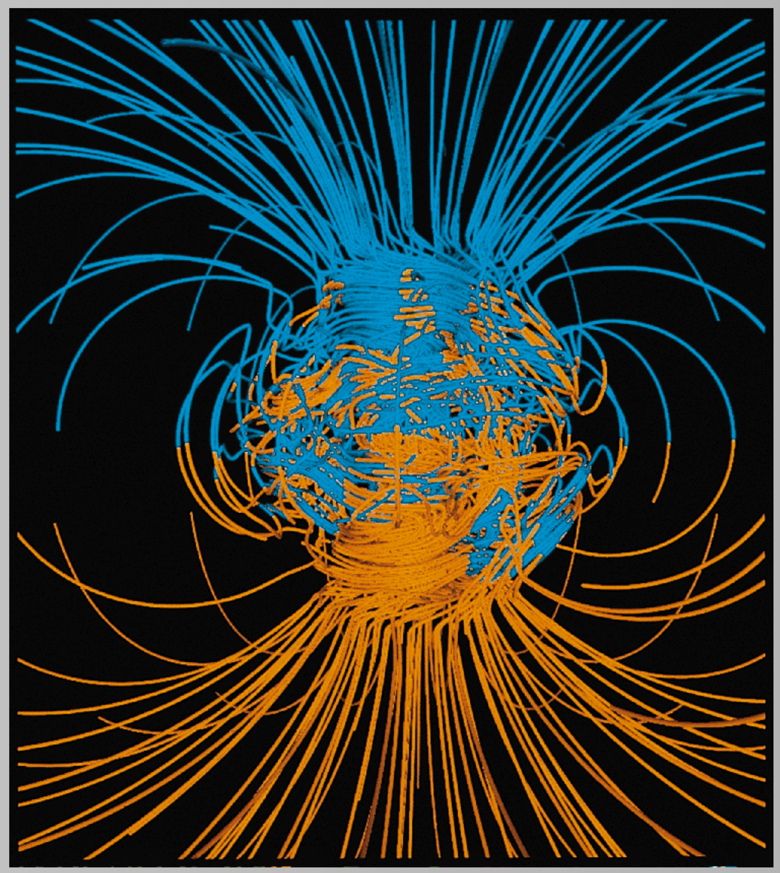

Figure I: A diagram of the magnetic field of the Earth. Note how quickly it vanishes as distance increases. [7.]

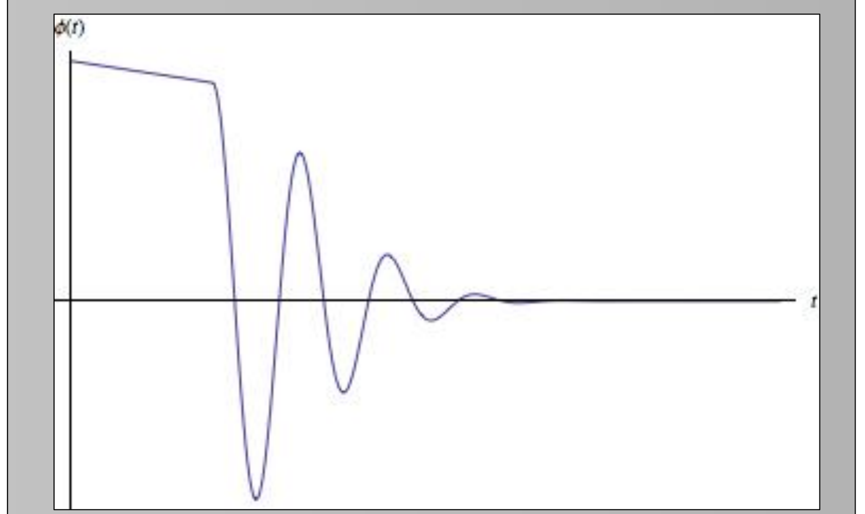

Figure II: A idealized diagram of the inflaton's behavior. It slowly decreases and then experiences extreme sinusoidal decay. The units of both axes are arbitrary.

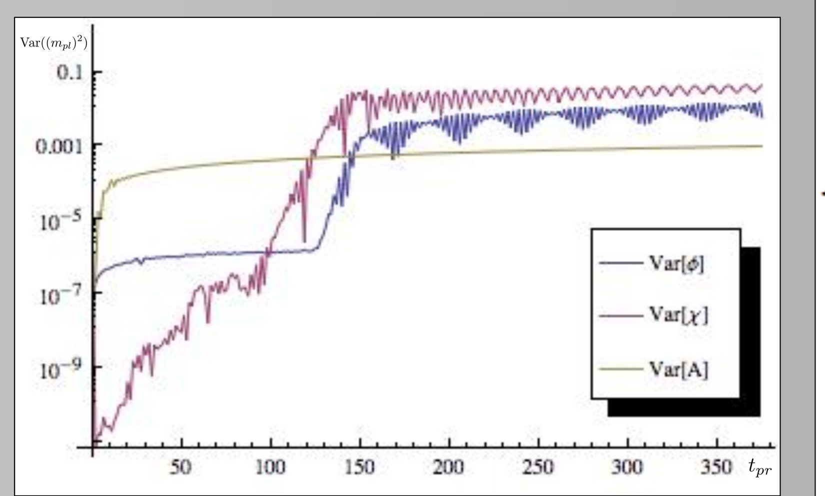

Figure III: Variances for our fields with respect to time. We have normalized our variances such that $\langle \Delta\phi^2 \rangle = a^3\langle \phi^2 \rangle - a^3\langle \phi \rangle^2$.

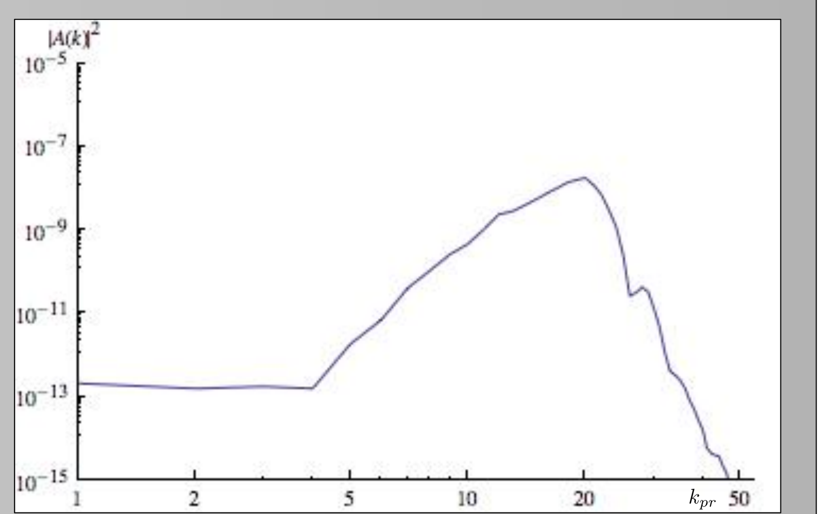

Figure V: The power spectrum for the 4-potential after 55 oscillations of the inflaton field.

Electrodynamics and the 4-Potential

We can define a field called the electromagnetic 4-potential that contains all the information about the magnetic fields of our model Universe. This potential is described by the electromagnetic action:

$$S = -\int \sqrt{-g}\,\mathrm{d}^4x\left[\frac{1}{16\pi}F_{\mu\nu}F^{\mu\nu} - A_\mu J^\mu\right]$$

This action is conformally invariant, which means that it will not independently generate the sorts of magnetic fields we wish to induce. To solve this problem, we will couple the 4-potential to our inflaton field. We use a coupling of the form:

$$V(\phi) = \frac{1}{4}f(\phi)F_{\mu\nu}F^{\mu\nu}$$

We can use this action to acquire the equation of motion for the 4-potential:

$$\Box^2 A_i + \frac{1}{4}f'(\phi)\,\dot{\phi}\,\epsilon_{ijk}\partial_j A_k = 0$$

Parametric Resonance and Preheating

Parametric resonance is a phenomenon observable both in classical and cosmological configurations. A common classical example is as follows: Imagine a child on a swing who periodically stands and squats to drive her motion. The frequency and amplitude of the swing's motion will vary based on the child's movement. We observe parametric resonance in coupled fields: parametric resonance between the inflaton and other fields drives preheating. We can describe parametric resonance with the Mathieu equation. For the χ field, the Mathieu equation is given as

$$\chi_k'' + (A_k - 2q\cos(2z))\chi_k = 0,$$

where z is proportional to time, and q and A_k vary slowly with time.

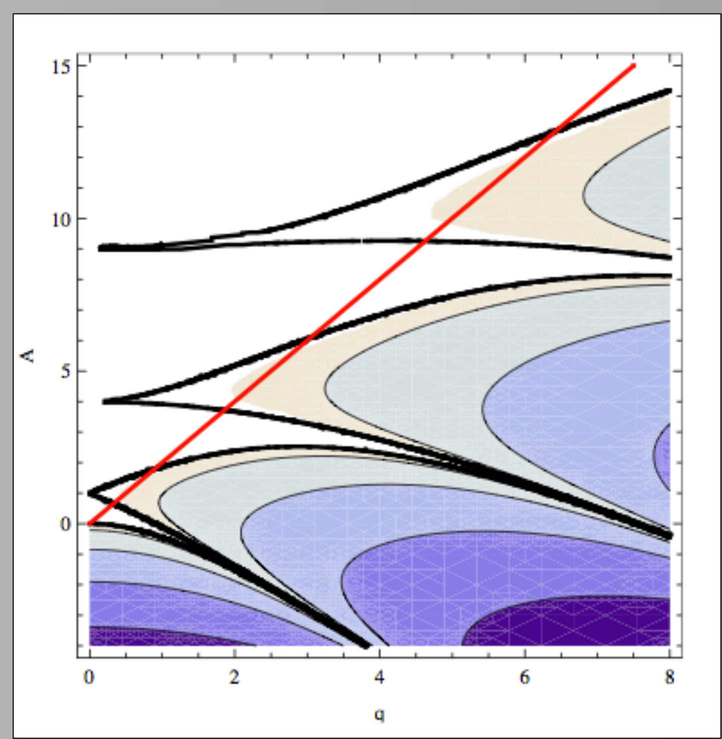

Figure IV: The Mathieu Stability and Instability Diagram. [5.] Solutions to the Mathieu equations are either stable (oscillatory) or unstable (exponential). This diagram shows which of these will be the case, given values of q and A.

Results and Looking Ahead

Fields that experience parametric resonance exhibit two noticeable characteristics. First, their variances initially increase rapidly with time and then remain fairly constant. And their power spectra witness a transition from low k-modes to high k-modes. Figure III gives our variances. As regards the ϕ and χ fields, the variances closely adhere to results from other studies. Thus, we can see that parametric resonance has taken place in our model. The variance for the 4-potential is slightly more peculiar, but its shape similarly suggests that some parametric resonance has been induced. Figure V gives the power spectrum for the 4-potential. As desired, it is dominated by high k-modes. We may conclude that, as a proof of concept, our hypothesis is not unviable. For further investigation, we would wish to more thoroughly examine the implementation of the 4-potential and other fields to ensure that it accurately represents our full cosmological knowledge. We would want also to extend the timeframe of our model such that it would be capable of making predictions about the modern state of intergalactic magnetic fields.