Metabolism, Microvilli, and the *Manduca sexta* Midgut: A Mathematical Model

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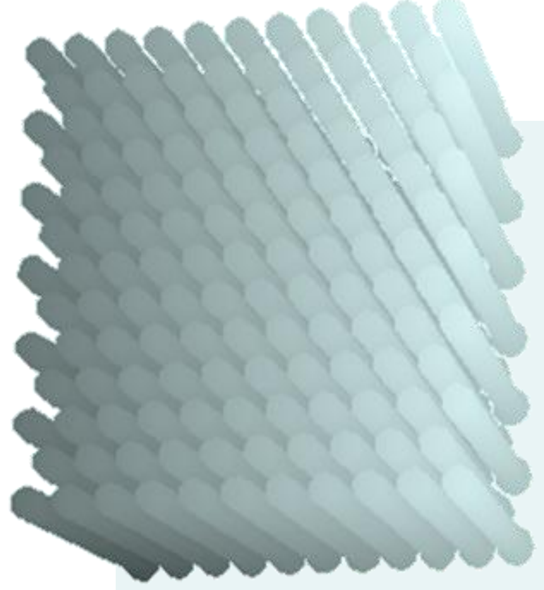


Abstract

Metabolism is the process by which energy obtained through food is used and stored and, for reasons unknown, scales with body weight consistently across species. *Manduca sexta*, a type of caterpillar which grows to maturity in only 18 days and exhibits a 10,000-fold increase in weight, is an ideal organism for studying this scaling of metabolism. It has been suggested that the surface area of the caterpillar's midgut may play a crucial role in metabolic scaling. We present a model which reflects the contribution of long, thin, finger-like structures called microvilli to *Manduca sexta* midgut surface area, compare our model to an existing model, and discuss applications of our model.

Introduction

Though scientists have shown that metabolic rate scales with body weight across species (Kleiber, 1932), it remains unclear *why* this occurs(Figure 1).

One possible explanation proposes a link between the amount of surface area available for energy-using processes and metabolic rate (Sernetz *et al*. 1985; West *et al*. 1999); the existence of such a link may explain the scaling of metabolism with body weight.

The current study is part of a larger project, the "Manduca InSTaRs (Interdisciplinary Science Training and Research)" project, which investigates metabolic scaling in a single, well-studied model organism: the *Manduca sexta* larva. These organisms are ideal for studying metabolism because while there is no marked change in behavior and morphology as the larva progress through five instars, they experience a 10,000-fold increase in weight in only 18 days (Goodman *et al.* 1985). These characteristics allow investigators to study a single larva over a wide range of sizes in a relatively short period of time.

Metabolic activity in *Manduca sexta* occurs in the highly folded midgut which displays more details which contribute to more surface area as magnification increases (Figure 2). We present a model of Microvilli, which can be seen at the highest magnification, contribute to the most to surface area (Wilson, 1962) and exhibit a constant shape across species.

Midgut sections can be sliced to give two different views of microvilli which show that microvilli have rounded tops and that microvilli bases curve into the midgut epithelium (Figure 3).

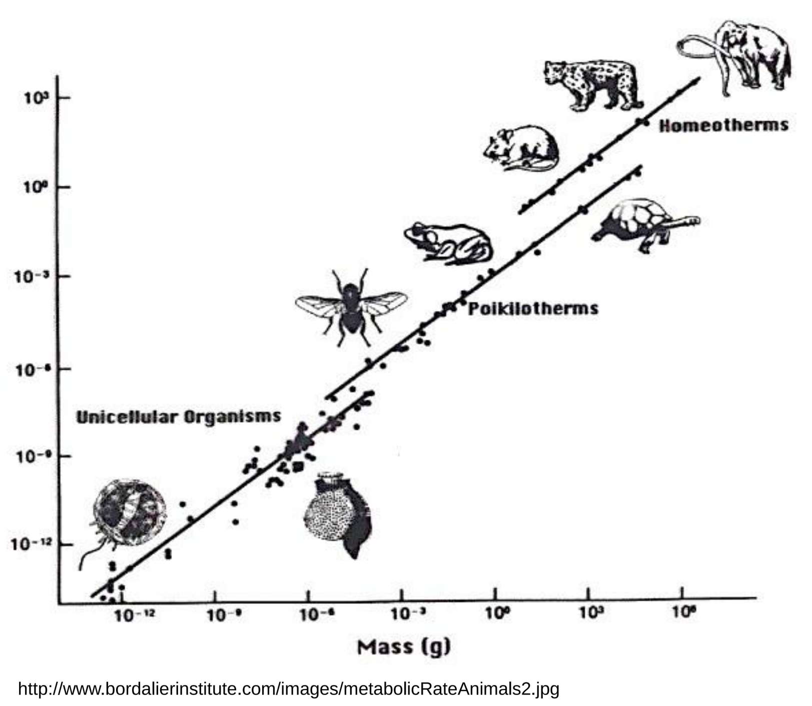


Figure 1 Metabolic rate scales with body weight across species. Each line shown here has a slope of 0.75.

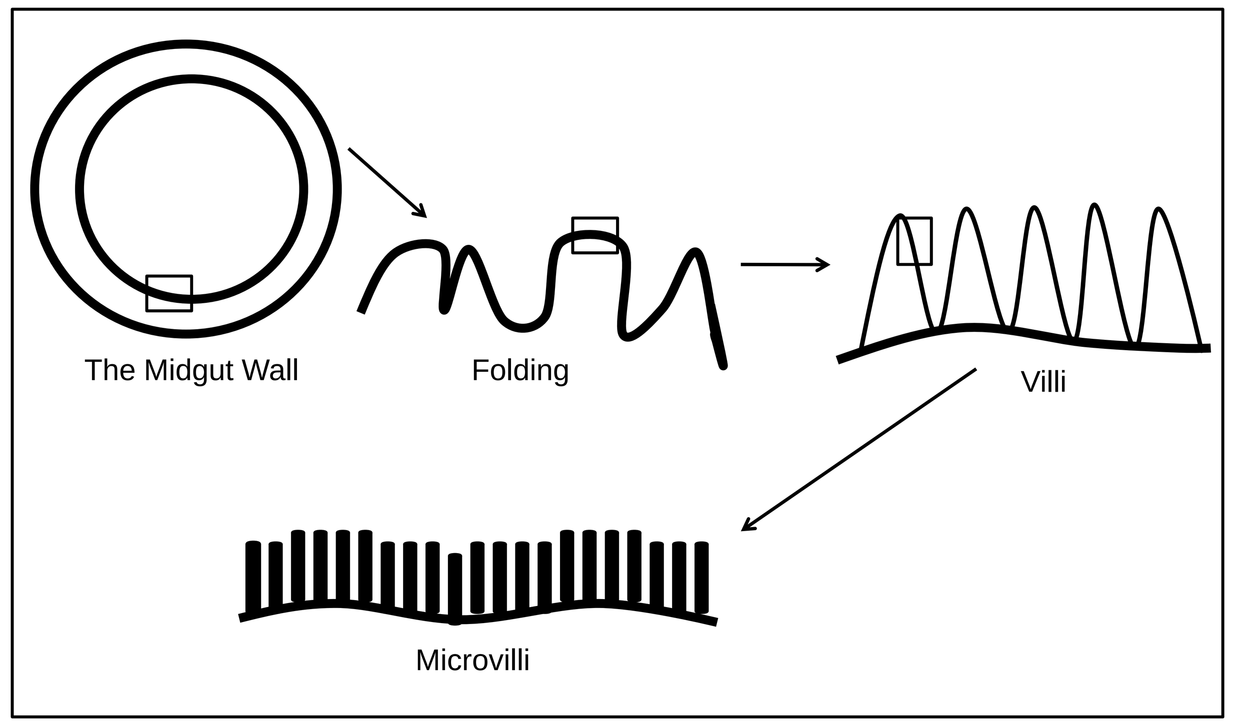


Figure 2 We can control the amount of detail we see in the midgut by adjusting the magnification. At high magnifications, we see microvilli, which contribute the most to surface area.

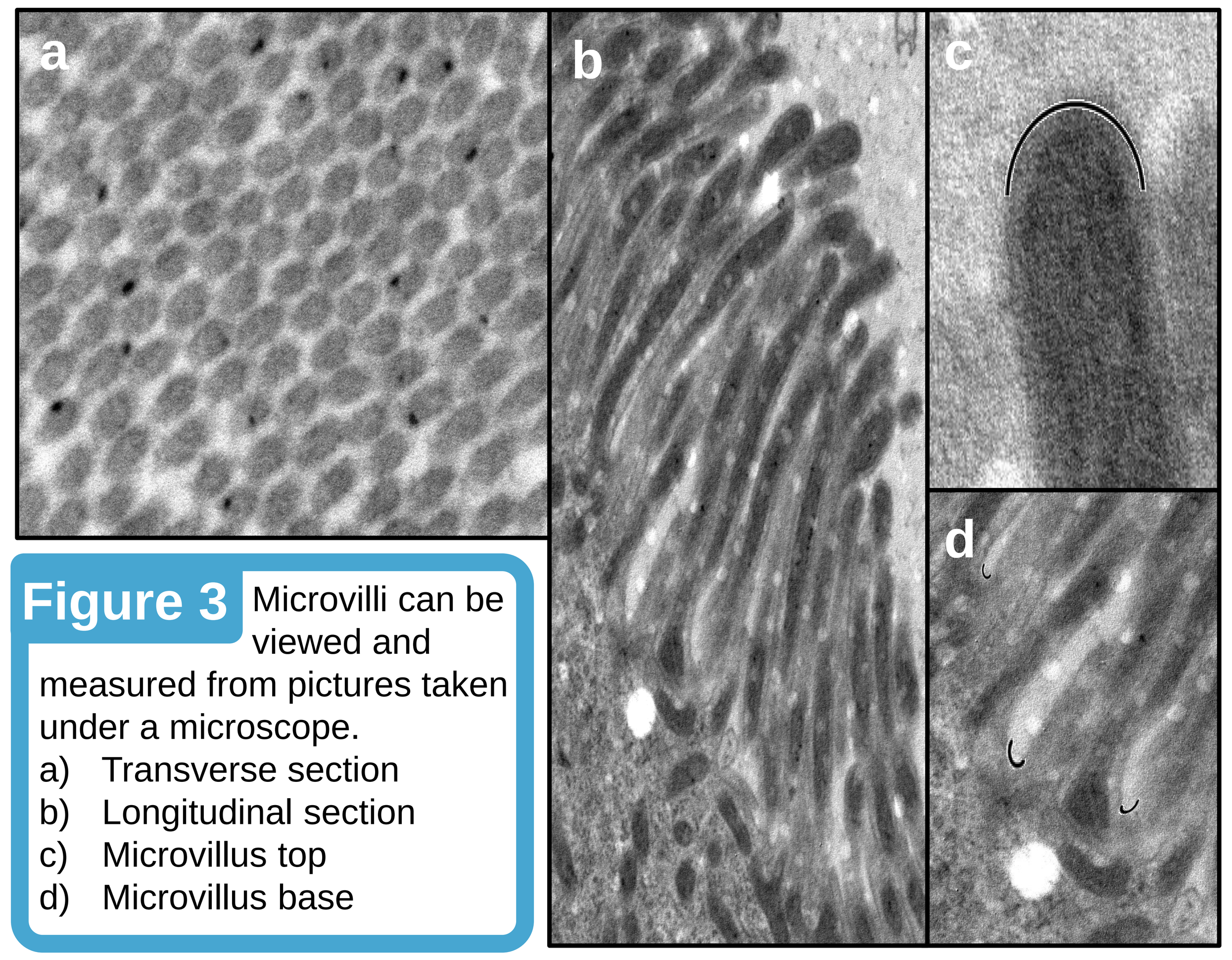
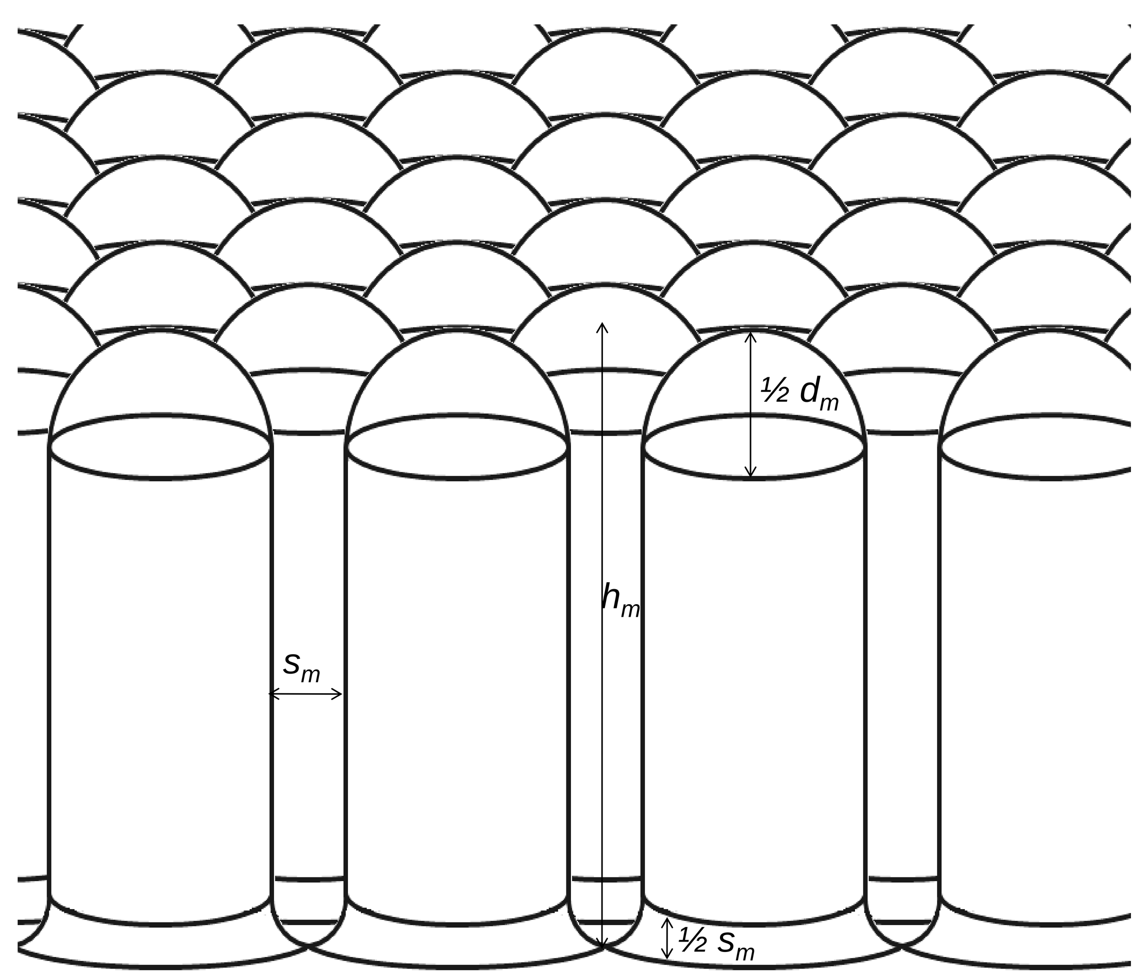


Figure 3 Microvilli can be viewed and measured from pictures taken under a microscope.
a) Transverse section
b) Longitudinal section
c) Microvillus top
d) Microvillus base

The Big Picture

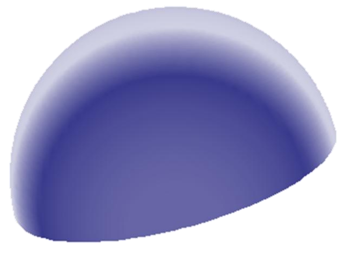
$$A_m = A_p + N_m(A_s + A_c + A_b)$$



The Three Pieces

We divide each microvillus into three simple shapes and find the surface area of each.

The Hemisphere



Radius = r = ½ d_m

Surface Area = ½ $4\pi r^2$
= $2\pi r^2$ = $2\pi(½\, d_m)^2$
= ½ $\pi d_m^2 = A_s$

The Cylinder

Radius = r = ½ d_m
Height = h = h_m – ½ d_m – ½ s_m
Surface Area = $2\pi rh$
= $2\pi(½\, d_m)(h_m – ½\, d_m – ½\, s_m)$
= $\pi d_m h_m – ½\, \pi d_m^2$
– ½ $\pi d_m s_m = A_c$

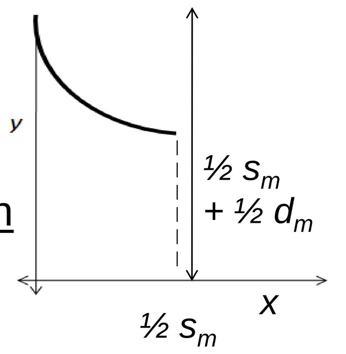
Expressing s_m in terms of d_m and N_m and simplifying gives

$$A_c = \pi d_m h_m - \frac{1}{2}\pi d_m \sqrt{\frac{2\sqrt{3}}{3N_m}}$$

The Annular Base

The Quarter Circle

$$c_m = -\sqrt{\left(\frac{1}{2}s_m\right)^2 - \left(x - \frac{1}{2}s_m\right)^2} + \frac{1}{2}d_m + \frac{1}{2}s_m$$



Surface Area of the Surface of Revolution

$$A_b = \int_0^{\frac{1}{2}s_m} 2\pi c_m \sqrt{1 + \left(\frac{dc_m}{dx}\right)^2}\, dx$$

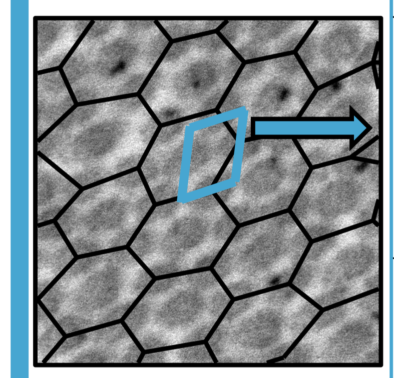
We can substitute c_m into A_b and assuming $s_m \geq 0$ we can evaluate the integral and simplify:

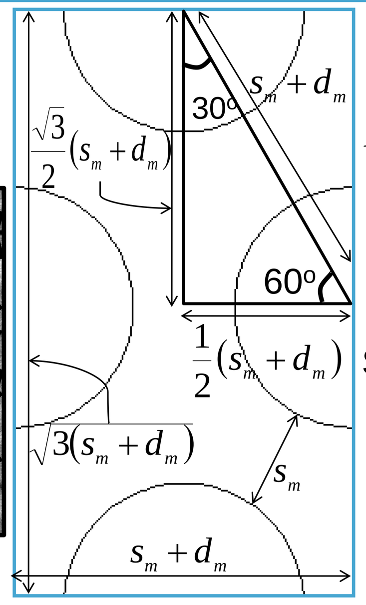
$$A_b = \frac{1}{4}\pi^2 s_m^2 - \frac{1}{2}\pi s_m^2 + \frac{1}{4}\pi^2 d_m s_m$$

Expressing s_m in terms of d_m and N_m and simplifying gives

$$A_b = \left(\frac{1}{4}\pi^2 - \frac{1}{2}\pi\right)\left(\frac{1}{\sqrt{N_m}}\sqrt{\frac{2\sqrt{3}}{3}} - d_m\right)^2 + \frac{1}{4}\pi^2\left(\frac{1}{\sqrt{N_m}}\sqrt{\frac{2\sqrt{3}}{3}} - d_m\right)d_m$$

Finding s_m





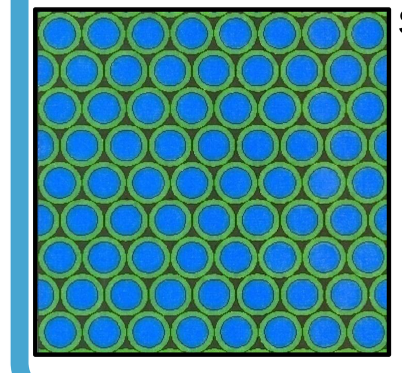
$$N_m = \frac{\#\ \text{Microvilli in the Region}}{\text{Area of the Region}} = \frac{2}{\sqrt{3}(d_m + s_m)^2}$$

Solving for s_m yields:

$$s_m = \frac{1}{\sqrt{N_m}}\sqrt{\frac{2\sqrt{3}}{3}} - d_m$$

The Flat Space (A_p)

The flat space is the portion of the midgut not covered by microvilli. It is black in the figure below.

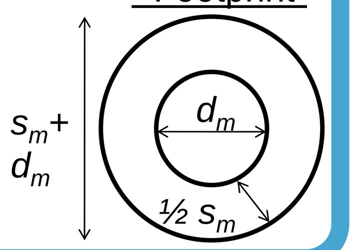


Surface Area of the Flat Space = A_p
= Area of the Square Planar Region
– (Area of a Microvillus "Footprint")
(Microvilli Density)

$$= 1 - N_m \pi\left(\frac{1}{2}d_m + \frac{1}{2}s_m\right)^2$$

$$= 1 - \frac{\sqrt{3}}{6}\pi$$

A Microvillus "Footprint"



Finding the Surface Area

$$A_m = A_p + N_m(A_s + A_c + A_b)$$

$$= 1 - \frac{\sqrt{3}}{6}\pi + N_m\left(\frac{1}{2}\pi d_m^2 + \pi d_m h_m - \frac{1}{2}\pi d_m\sqrt{\frac{2\sqrt{3}}{3N_m}} + \left(\frac{1}{4}\pi^2 - \frac{1}{2}\pi\right)\left(\frac{1}{\sqrt{N_m}}\sqrt{\frac{2\sqrt{3}}{3}} - d_m\right)^2 + \frac{1}{4}\pi^2\left(\frac{1}{\sqrt{N_m}}\sqrt{\frac{2\sqrt{3}}{3}} - d_m\right)d_m\right)$$

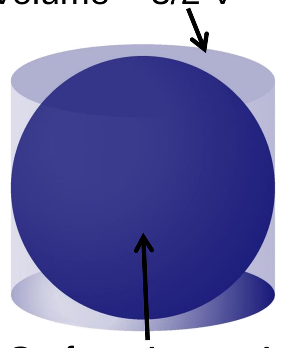
Simplifying yields:

$$A_m = 1 + \pi\left(d_m h_m N_m + \left(\frac{1}{2} - \frac{\pi}{4}\right)d_m\sqrt{\frac{2\sqrt{3}}{3}N_m} + \frac{\sqrt{3}\pi - 3\sqrt{3}}{6}\right)$$

where A_m is the surface area of a 1 square μm region of midgut covered by N_m microvilli with diameter dm and height h_m.

Archimedes and Microvilli?

Surface Area = 3/2 A
Volume = 3/2 V



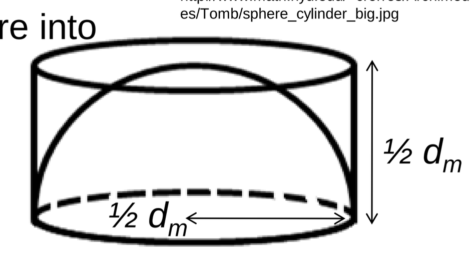
Surface Area = A
Volume = V

Picture Above from: http://www.math.nyu.edu/~crorres/Archimedes/Tomb/sphere_cylinder_big.jpg

Archimedes discovered that a right circular cylinder has one and a half times the surface area and volume of the sphere inscribed inside (Sherman 1999, top right).

Taking half of this figure and removing the top gives a hemisphere and a topless right circular cylinder with the same surface area (bottom right).

Using this, we can combine the top two portions of a microvillus, the cylinder and the hemisphere into one topless cylinder, the surface area of which is given by,



$$A_t = 2\pi rh = 2\pi\left(\frac{1}{2}d_m\right)\left(h_m - \frac{1}{2}s_m\right)$$

$$= \pi d_m h_m - \frac{1}{2}\pi d_m s = \pi d_m\left(h_m - \frac{1}{2}\sqrt{\frac{2\sqrt{3}}{3N_m}} + \frac{1}{2}d_m\right) = A_s + A_c$$

Applications of the Model

- With enough data, we can construct functions, $h_m(x)$, $d_m(x)$, and $N_m(x)$,(where x = time, location, body weight, etc.).
- This allows us to examine how surface area changes with x.
- Our model can also be used to find microvilli surface area in other organisms.
- We can also find the surface area of larger, non-microscopic projections.

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Acknowledgements

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