Finding an Upper Bound on the Angular Momentum of a Neutron Star

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Abstract:

A neutron star is one of the densest states of matter in the universe. The behavior of matter at low temperatures and densities greater than the atomic nucleus is currently not completely understood. The behavior of matter is governed by both the forces acting on it and the intrinsic properties of the matter itself. These intrinsic properties of the matter can be related to one another, mathematically, through an *equation of state,* which expresses the relationship between the matter’s pressure, density and temperature. Since the equation of state governs the physically observable properties of a neutron star, by placing limits on these, for a given equation of state, it is possible to discover a more precise equation of state for high density matter.

Neutron Stars:

In1967 by Jocelyn Bell observed a rapidly pulsating radio signal. Although it was initially believed to be from extraterrestrials, it was later found to be a rapidly rotating neutron star.

When a massive star expends its fuel supply, it implodes in a supernova. After the supernova, all that is left of the star is its crushed core, which forms a neutron star. A neutron star typically has a more mass than the Sun, within a 10 km radius. Consequently, a neutron star is incredibly dense – denser than an atomic nucleus. Another interesting consequence of this highly compactified matter , is that the constituent particles are frozen in place, so a neutron star, can be thought of as having a low temperature.

Angular Momentum:

Just as momentum quantifies the tendency of an object to keep moving, angular momentum quantifies the tendency of a spinning object to keep spinning.

$$L = I\omega = |\vec{r} \times \vec{p}|$$

Here *L* is angular momentum, *I* is moment of inertia, and ω is angular velocity (revolutions per second).

The angular momentum of a top enables it to balance itself against gravity.

Methodology:

The main goal of this project was to find the largest values of the angular momentum of a neutron star, for a given equation of state.

The angular momentum of a neutron star was computed with a computer program. The program calculated the angular momentum of a neutron star at different angular velocities for a specified central density and equation of state.

This was done by numerically integrating Einstein’s equations (of gravity) over the mass distribution produced by a given equation of state with a specified central density, for various angular velocities.

The largest value of angular momentum attainable at each central density was plotted against the mass of such a star, and this was repeated for a number of different equations of state.

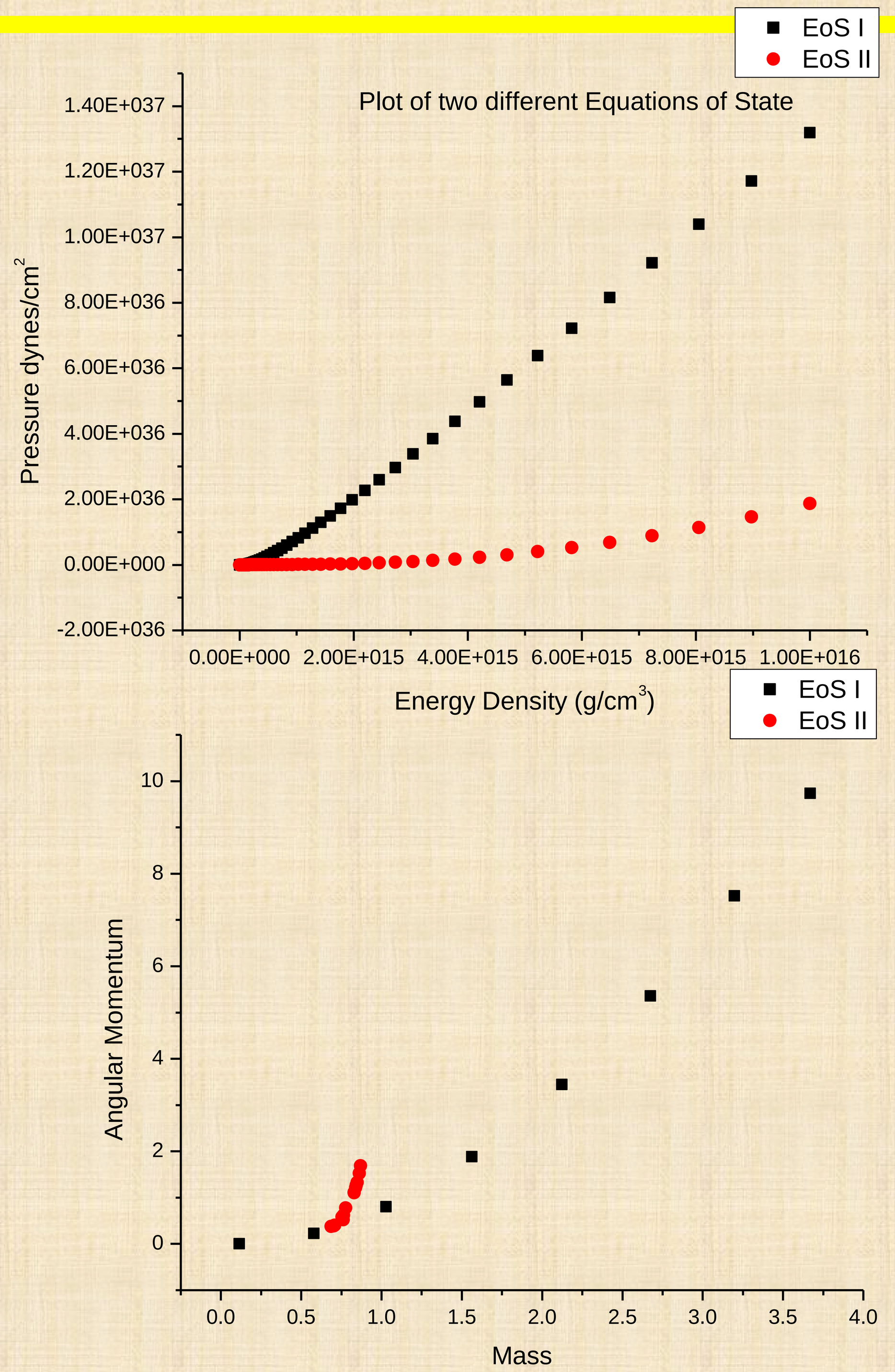

Figure I: The first plot shows two different equations of state. The second shows the maximum angular momentum each equation of state can produce as a function of mass.

Hypothesis:

Using a Newtonian approximation, of a sphere of uniform density it was found that

$$L \propto \sqrt{MR^3}$$

Since a stiffer equation of state (EoS I in fig I), is capable of supporting more mass, and a large radius, it was initially assumed that the maximum angular momentum would be generated by the stiffest equation of state.

As Figure I demonstrates, EoS I is stiffer and is capable of producing stars with greater masses and angular momentum.

EoS II on the other had is softer, and although it cannot reach the highest masses, it can produce a bigger angular momentum for a given mass than EoS I, repudiating the initial hypothesis.

This suggests that the mass of a neutron star with a soft equation of state is distributed farther from the center than the mass of a neutron star with a stiff equation of state.

Bounds on the Equation of State:

The equation of state must be within the limits of the known laws of physics.

Since the speed of sound in a neutron star cannot exceed the speed of light, the equation of state’s slope (the speed of sound squared) must be less than the speed of light squared.

The equation of state must be at least as stiff as less dense matter.
The equation of state must be within these parameters.

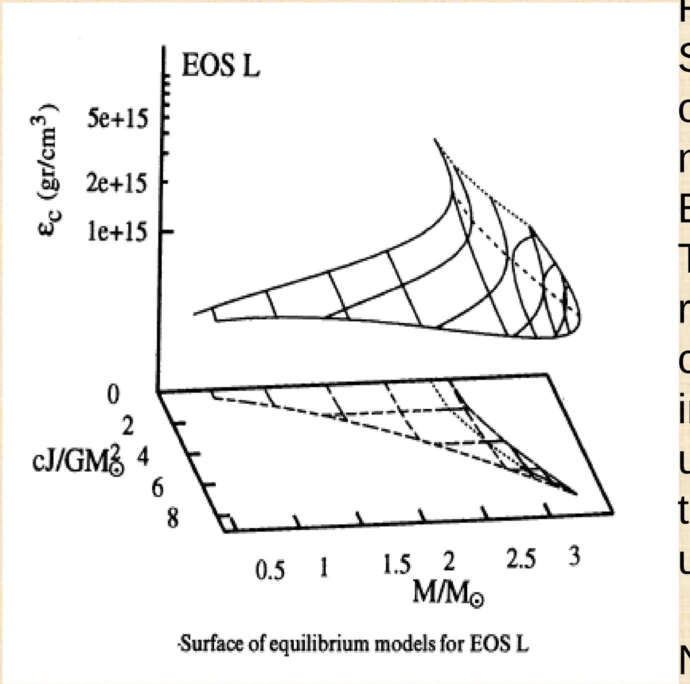

Figure 2: This plot shows Space of the parameters of physically possible neutron stars with a given Equation of sate.
The diagonal edge represents instability caused by rotational instability, while the upward curve is where the mass becomes unstable.

Note that the angular momentum reaches a peak value.

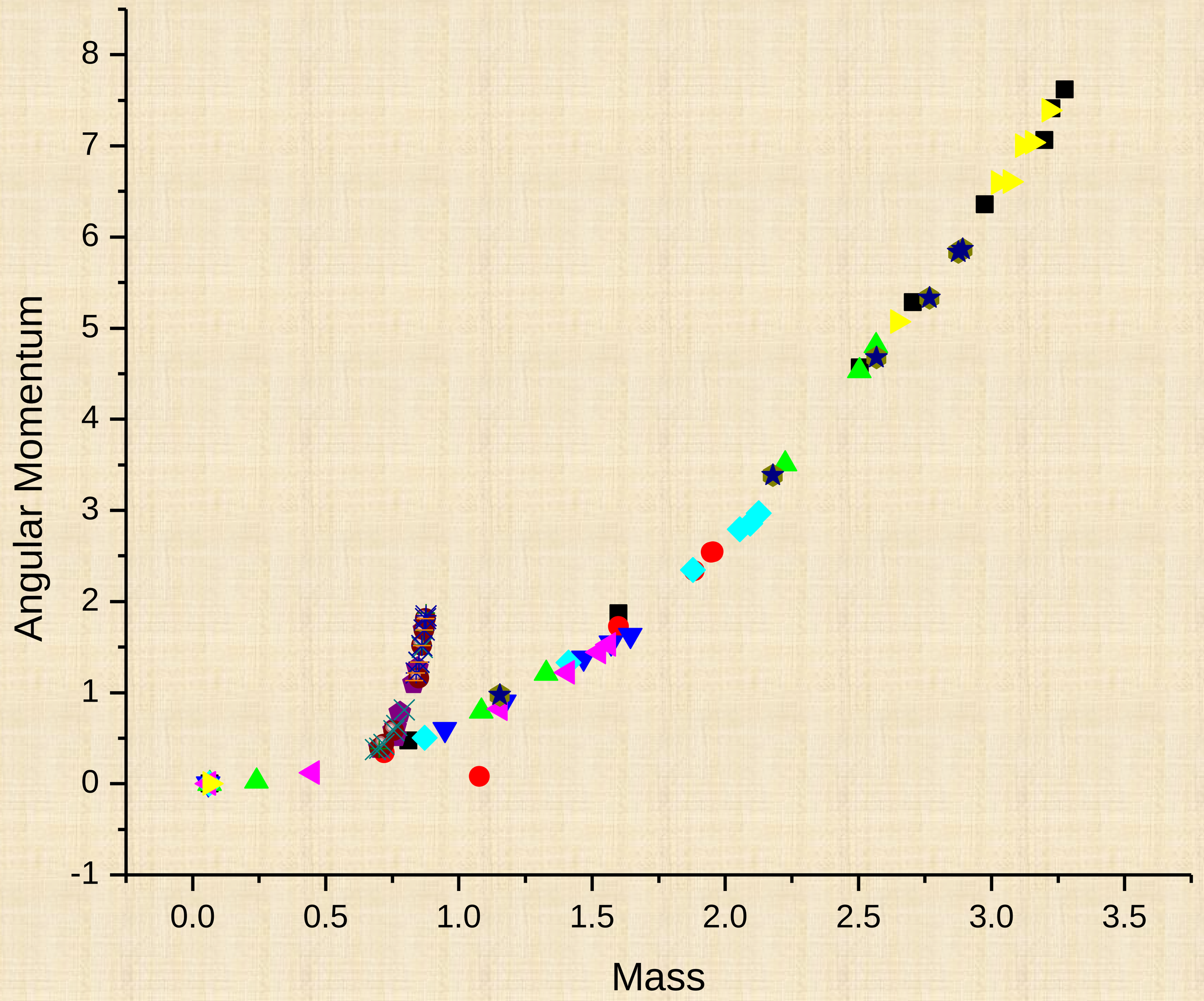

Figure 2 The above plot shows a compilation of the angular momentum plots for numerous equations of state.

Results:

The absolute maximal angular momentum was

$$8.21 \quad km^2 \cdot SolarMasses/s$$

This however occurred at 3.6 solar masses, which is unrealistic since the most massive observed neutron star is 2.1 solar masses.

In the Regime below the largest observed mass, the maximal angular momentum is

$$2.48 \quad km^2 \cdot SolarMasses/s$$

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