Seeding the Large-Scale Structure of the Universe: Cosmological Perturbations from Staggered Inflation

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ABSTRACT

The current detailed picture of the Cosmic Microwave Background (CMB) and our observations of the large-scale structure of the Universe give measurable hints about how our Universe formed. The theory of cosmological perturbations is a promising explanation for the origin of structure in the context of inflationary cosmology. This research looks at how the small quantum fluctuations due to the Bunch-Davies vacuum state at the outset of inflation could source the clusters and galaxies that exist throughout the Universe. Key to tracking these perturbations is the evolution of the cosmological scale factor, $a(t)$, through the inflationary epoch. This is the first objective of this research: to evolve the scale factor with potential-driven inflation from multiple scalar fields. This model is referred to as staggered inflation, or *N*-flation. The combined potential energy from the fields provides the energy needed to create the near-exponential growth of the Universe. To mirror theoretical expectations the scale factor must increase by a factor of e^{60} in our simulations of inflation. The second objective is to evolve the perturbations. To test the viability of the models, we perform spectral analysis of the perturbation modes. This results in a power spectrum of curvature perturbations that can be used in comparisons with data from observations of the CMB.

FLRW METRIC

The system of measurement used to describe the distance between points in the Universe is called a metric. The metric used in our models is the Friedmann-Lemaître-Robertson-Walker (FLRW) metric. The most general form of this metric is

$$g_{\mu\nu} = \begin{pmatrix} -c^2 & 0 & 0 & 0 \\ 0 & a^2(t) & 0 & 0 \\ 0 & 0 & a^2(t)S_\kappa^2(r) & 0 \\ 0 & 0 & 0 & a^2(t)S_\kappa^2(r)\sin^2\theta \end{pmatrix}, \text{ where } S_\kappa(r) = \begin{cases} R\sin(r/R) & (\kappa=+1) \\ r & (\kappa=0) \\ R\sinh(r/R) & (\kappa=-1) \end{cases}$$

however, we consider the case in which the Universe is spatially flat. This simplifies the line element of the FLRW metric to

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -c^2dt^2 + a^2(t)\left(dx^2+dy^2+dz^2\right).$$

The features of the metric necessarily reflect those of cosmological principle, which states that our Universe is homogeneous and isotropic. There is no special position or preferential direction and that manifests itself in the FLRW metric in which the scale factor depends on time alone and scales each of the three spatial dimensions equally.

EQUATIONS OF MOTION

The scale factor, $a(t)$, is time-dependent, and since it scales the spatial distances in the Universe, knowing how it changes with time explains how Universe evolves. The equations of motion for $a(t)$ are derived from general relativity (first done by Alexander Friedmann). The first Friedmann equation is

$$H^2 = \left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}\rho\,.$$

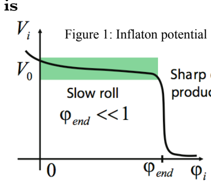

Figure 1: Inflaton potential

Because our models of inflation are potential-driven and invoke many fields, the energy density, $\rho(t)$, appearing in the Friedmann equation is a sum of the total energy of each scalar field. Each field, referred to as an *inflaton*, has energy density, ρ_i :

$$\rho_i = \frac{1}{2}\dot\varphi_i^2 + V(\varphi_i)\,.$$

The inflatons are time-dependent; different fields will dominate the energy density at different times during inflation. With the scale factor evolution dependent on the energy density and the energy density evolution dependent on the fields, we are left with these coupled differential equations that we can evolve numerically:

$$\ddot\varphi_i + 3\frac{\dot a}{a}\dot\varphi_i = -\frac{dV}{d\varphi_i} \quad \text{and} \quad \left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}\left[\sum_i^N\left(\frac{1}{2}\dot\varphi_i^2 + V(\varphi_i)\right) + \rho_{rad}\right].$$

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PERTURBATIONS

We consider scalar perturbations that are a linear-order expansion about the FLRW metric.

$$\bar g_{\mu\nu} = g_{\mu\nu} + \delta g_{\mu\nu}$$

In the most general case of scalar perturbation theory there are four free parameters. By choosing the Newtonian gauge and considering models with no anisotropic stress, we reduce the description of the perturbations to a single variable, *Q*, the Newtonian potential, and incorporate another equation of motion into our simulations,

$$\ddot Q_I + 3H\dot Q_I + \frac{k^2}{a^2}Q_I + \sum_J^N\left(\frac{\partial^2 V}{\partial\varphi_I\partial\varphi_J} - \frac{8\pi G}{a^3}\frac{d}{dt}\left(\frac{a^3}{H}\dot\varphi_I\dot\varphi_J\right)\right)Q_J = 0.$$

The subscript index, *I*, indicates that there is a component of the Newtonian potential that corresponds to each inflaton. Additionally, each Q_I is decomposed by a Fourier transform into many modes with corresponding wavenumbers, *k*. The evolution of the perturbations is dependent on both the scale factor and the inflatons, however, the evolution of the scale factor and inflatons is unaffected by perturbations. Each perturbation mode is initialized to the conditions subscribed by the Bunch-Davies vacuum state. The Bunch-Davies initialization for each of the modes of the perturbations is

$$Q_I = \frac{1}{a\sqrt{2k}}\left(\alpha_I e^{-ik\tau} + \beta_I e^{ik\tau}\right).$$

To interpret the effect of perturbations in our models of the Universe, we look at the spatial curvature perturbation, *R*:

$$R = \frac{1}{3(1+w)\,m_{pl}^2\mathcal{H}}\sum_I \phi_I' Q_I.$$

SIMULATIONS AND TENTATIVE RESULTS

The decomposition of the second-order differential equations of motion for the scale factor, inflatons and perturbations into a set of coupled first-order equations enables us to solve them numerically. We employ the fifth-order Cash-Karp Runge-Kutta integration technique which has an adaptive time-step. As the program steps through time, the inflatons decay and drop out one by one as seen in Figure 4. Below are the resultant power spectra of curvature perturbations at the end of inflation simulations with different numbers of inflatons.

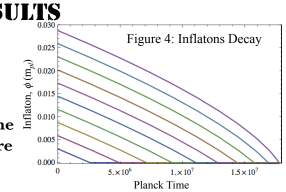
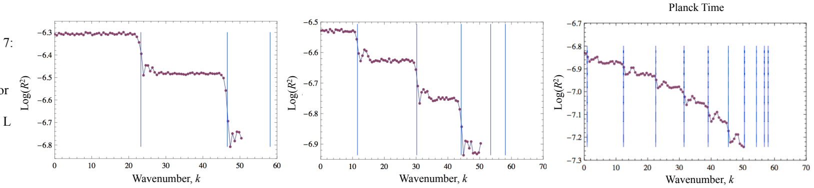


Figure 4: Inflatons Decay



Figures 5, 6, & 7: Spectrum of curvature perturbations for 3, 5, and 10 inflatons (from L to R)

FUTURE WORK

Our models must agree with the observations of the CMB. Therefore, we must create an angular power spectrum for our models. This will tell us if our effective model of staggered inflation would leave the same imprint of anisotropies on the CMB that we observe.

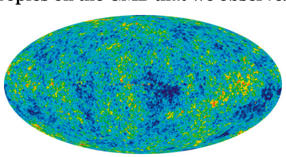


Figure 2: Temperature fluctuations, $\Delta T/T$, in the CMB are on the order of 10^{-5}. These provide important information on the dynamics of the early Universe. (NASA, 2010)

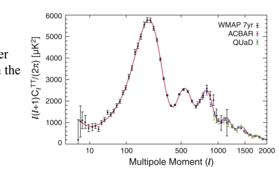


Figure 3: The angular power spectrum of the CMB from the 7-year WMAP results. (NASA, 2010)

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