Supernovae and Cosmic Anisotropy

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Abstract:

It is assumed that our Universe is isotropic, it expands in the same way in all directions, and homogeneous, it is the same everywhere. Under this assumption *dark energy*, a mysterious fluid that we do not understand, is needed to explain the expansion history of our Universe. This project explores anisotropic universes, universes that expand at different rates in different directions. Can the data be explained better with an anisotropic universe than one that includes dark energy?

Type Ia supernovae are considered to be standard candles, their luminosity when they go supernovae is known, and it is from them that we get the data that tells us about how our Universe expanded in the past. I wrote a computer program that evolves anisotropic models of the Universe. Monte Carlo analysis was used to see if the anisotropic Universe would have an expansion history which would be consistent with the data that we get from type Ia supernovae.

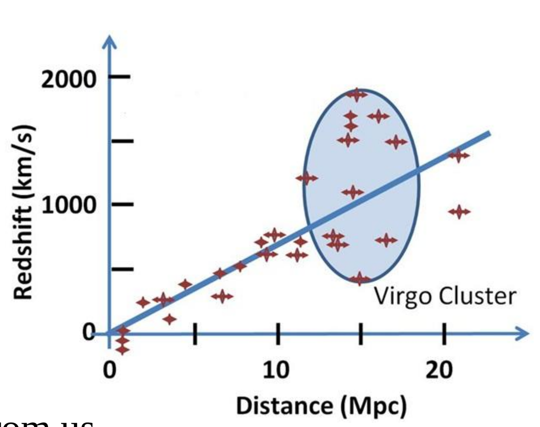
Supernovae:

Supernovae are the extremely bright and energetic explosions of massive stars. They can cause a burst of radiation that can outshine an entire galaxy. One lets off about 1 to 2×10^{44} joules of energy, which is about as much as energy as the Sun is expected to let off in its life time [1].

There are two main classes of supernovae, type I and type II. Type I supernovae occur in large stars which do not contain much hydrogen. Type Ia supernovae (SNIa) are a subclass of type I supernovae. Both Type I and type II supernovae can be caused either by the sudden turning on or off of the production of energy through nuclear fusion [2].

Nuclear fusion is the process during which multiple atomic nuclei join to form heavier nuclei. Depending on the nuclei that are joined, this process can either emit or absorb energy[3].

Hubble Diagram:



A Hubble diagram is a graph of the distances to versus the redshifts of objects in the sky and we use them to look at the data we have from supernovae [4]. The redshift of an object measures how quickly it is moving relative to us, and if it is moving towards or away from us.

The distance to an object in the sky is determined using the magnitude of the object, which describes how bright it is. The distance modulus is one way to describe the relative distance between two objects. The distance modulus is simply the difference between the apparent magnitude, *m*, of the supernova which we can observe and its absolute magnitude, *M*.

It is important that SNIa are able to be treated as standard candles. Being a standard candle means that we know how bright they are when they explode, and we need to know this in order to figure out the distance to them.

References:

[1] Khoklov, A., Mueller, E and Hoeflich, P., 1993, Astronomy and Astrophysics.
[2] Ignacio Ferreras and Joseph Silk, 2002, Astron. Soc.
[3] Arny, Thomas T., "Explorations: An Introduction to Astronomy". Custom Publishing, 2006.
[4] Wikipedia, "Hubble's Law", http://en.wikipedia.org/wiki/Hubble_diagram.
[5] Wikipedia, "Isotopes of Nickel", http://en.wikipedia.org/wiki/Nickel-56Nickel-56.
[6] Wikipedia, "Type 1a Supernova", http://en.wikipedia.org/wiki/Type_1a_supernova.

Why can we treat SNIa as standard candles?

A simple answer to this question is that all SNIa produce the same lightcurve, which is a graph of luminosity as a function of time, as can be seen in Figure 2. SNIa occur when white dwarfs reach the Chandrasekhar mass. When they reach this mass the electron degeneracy pressure within the star is no longer strong enough to oppose the stars collapse and so the star goes supernova, letting off a large amount of energy. Because all SNIa occur when white dwarfs have reached the same mass, they all let off the same amount of luminosity.

Nickel-56 is created as the star collapses and heats up. It is a very unstable element and,

$$Ni^{56} \rightarrow Co^{56} + e^{+} + \nu$$

via beta plus decay with a half-life of 6.077 days. Cobalt-56 is also unstable and,

$$Co^{56} \rightarrow Fe^{56} + e^{+} + \nu$$

with a half-life of 78 days. The energy that the beta decay processes use is what causes the luminosity of the supernovae to decrease [5].

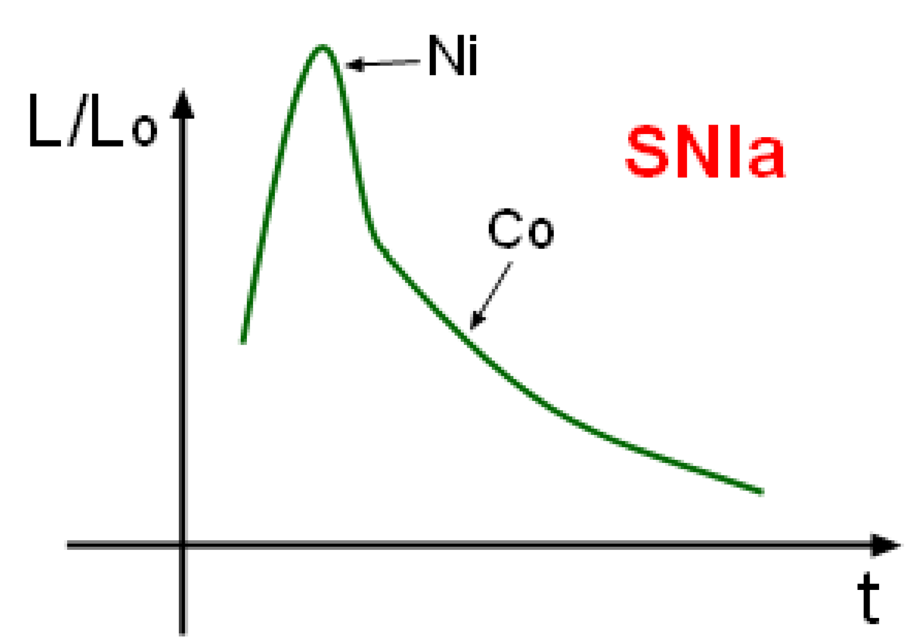


Figure 1: The lightcurve for SNIa. The initial decline in luminosity is caused by the decay of nickel-56 into colbalt-56 and the more gradual decline is caused by the decay of cobalt-56 into iron-56 [6].

Isotropic Universe:

The FLRW metric describes the geometry of an isotropic and homogeneous universe,

$$ds^2 = c^2 dt^2 - (dx^2 + dy^2 + dz^2) a(t)^2$$

where *a(t)* is the scale factor which describes the expansion of our universe.

Friedmann's equations are solutions to Einstein's equations and they relate the expansion of the universe with what is in it. They are,

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\rho \qquad \left(\frac{\ddot{a}}{a}\right) = -\frac{4\pi G}{3c^2}(\rho + 3P)$$

where *P* is pressure and ρ is the energy density, and these two quantities are related by the equation of state,

$$P = \omega \rho$$

Using these equations I solve for the acceleration of the scale factor,

$$\ddot{a} = -\frac{1}{2}\frac{\dot{a}^2}{a}(1 + 3\omega)$$

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The Program:

The equation for the acceleration of the scale factor is not analytically solvable. I wrote a program that solves for *a(t)* using Monte Carlo integration. For each run of the program it selects random initial conditions of the Hubble constant and the amount of matter that is in the Universe. Hubble's constant is given by,

$$H_0 = \frac{\dot{a}_0}{a_0}$$

and we define the scale factor now, a_0, to be 1 so by choosing Hubble's constant we have chosen the initial condition for the velocity of the scale factor.

The supernova data I use gives the redshift and distance modulus of different SNIa. Once the program solves for *a(t)* it finds the redshift at all times using the following equation,

$$z(t) = \frac{a_0}{a(t)} - 1$$

Once the program comes across a redshift that agrees with a redshift from the data it finds the luminosity distance at that time,

$$D_L(t) = -\frac{1}{a(t)}\int_{t_0}^{t} \frac{c}{a(t')}dt'$$

which is changed into the distance modulus by the following relationship,

$$D_m = m - M = 5(log D_L - 1)$$

Once the program finds the distance modulus, it compares it to the distance modulus from the data and a chi squared statistic is used to find the most likely set of initial conditions, aka the set of initial conditions that produce a lightcurve that agrees the most with the data.

Tentative Isotropic Results:

The program is run with a million different pairs of initial conditions. The most likely density of matter is $\Omega_m = 0.256 \pm 0.14$ and the most likely value of the dimensionless Hubble parameter is $h = 0.71 \pm 0.29$. This is the pair of initial conditions which have the highest likelihood, as can be seen in Figure 2. Both of these values agree with other studies on isotropic universes, and because of this we know that our program works and we can move on to the anisotropic case.

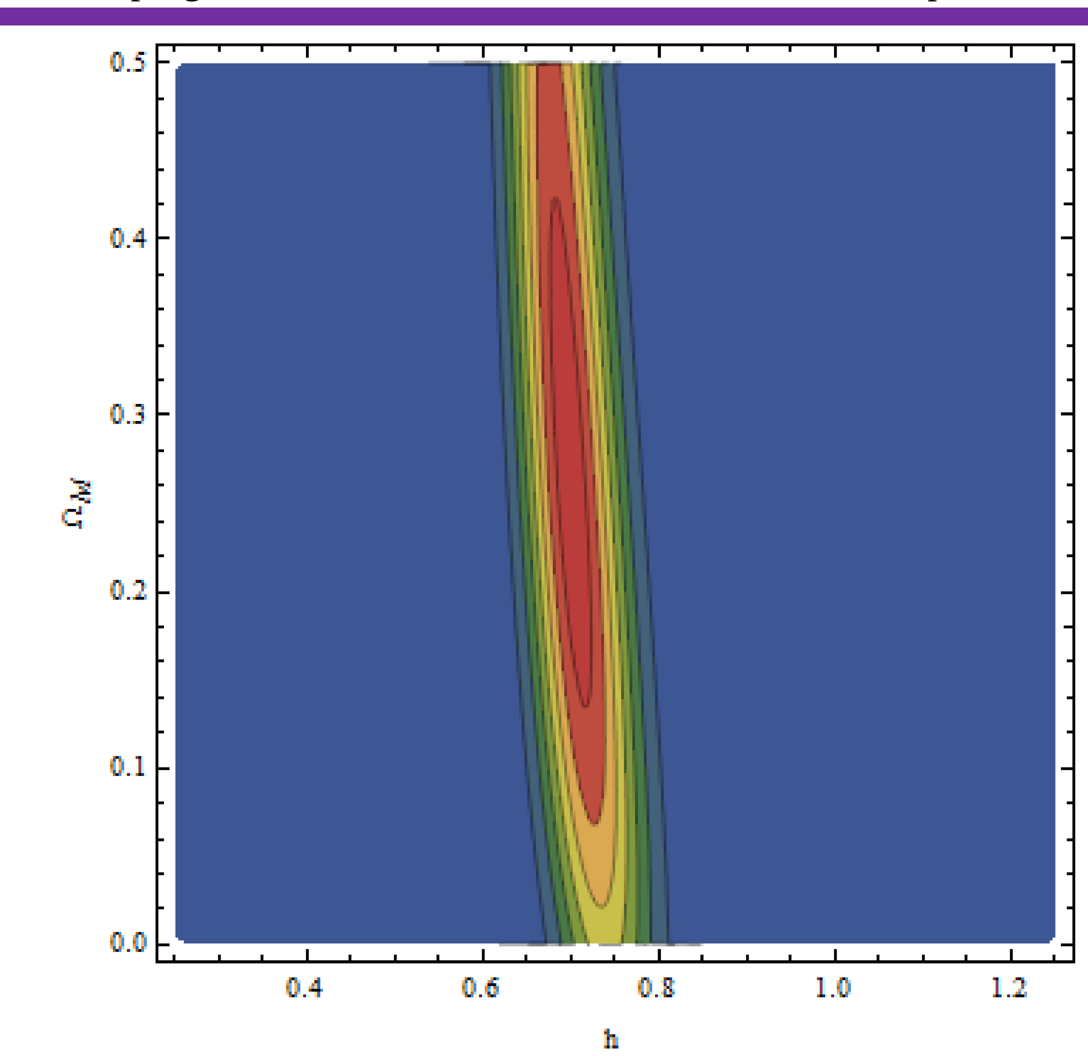

Figure 2: A plot of the likelihoods for one million different pairs of initial conditions.

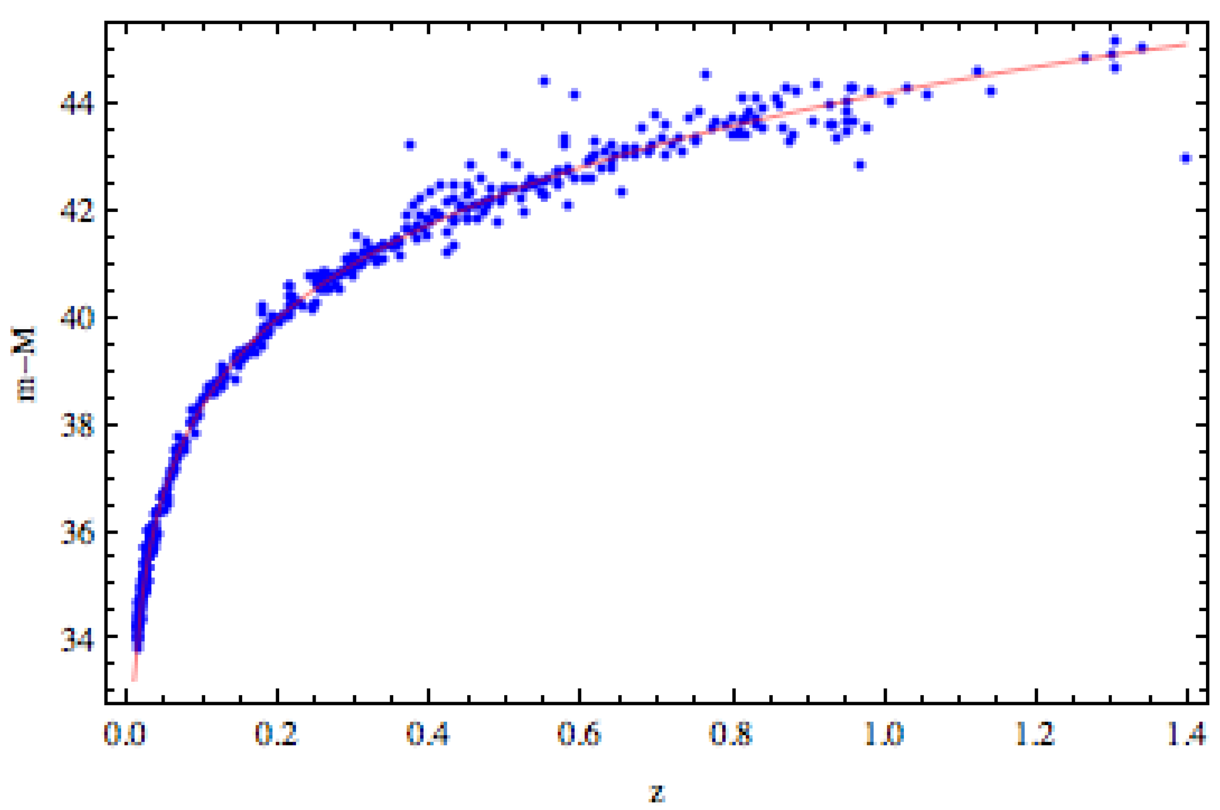

Figure 3: The blue points are the data we have from supernovae, and the red curve is the lightcurve that we get with parameter values of h =0.71 and Ω_m = 0.256.

Anisotropic Case:

Next we assume that the Universe is slightly anisotropic, meaning that it expands at different rates in different directions. The anisotropic universe we look at is described by the following metric,

$$ds^2 = c^2 dt^2 - dx^2 a(t)^2 - dy^2 b(t)^2 - dz^2 e(t)^2$$

where there are now three scale factors, *a(t)*, *b(t)* and *e(t)*, instead of just one, because each direction in an anisotropic universe expands at a different rate.

Now that we are considering an anisotropic universe the position of the supernovae in the sky is important. We change the equations for redshift and the distance modulus so that they depend on the position of the supernovae.

Einstein's equations are solved for the anisotropic case with the metric above and from it's solutions the acceleration of each scale factor is found. As in the isotropic case, the program selects random initial conditions for the density of matter in the universe, and for the Hubble constant in each direction, and from this *a(t)*, *b(t)* and *e(t)* are solved for.

Tentative Anisotropic Results:

After the program runs 2000 runs, each with its own set of initial conditions, we find that the most likely values of the dimensionless Hubble parameter in each direction are,

$h_a = .73 \pm .01$
$h_b = .73 \pm .01$
$h_e = .73 \pm .01$

which are all statistically the same, so we do not see evidence for anisotropy.

We found the density of matter to be

$\Omega_m = .26 \pm .27$.

We are hoping to find that Ω_m is close to 1. We assume that there is only matter and dark energy, Λ, in our Universe, so $\Omega_m + \Omega_\Lambda = 1$. Since $\Omega_m = .26 \pm .27$ this is saying that the density of dark energy in the Universe is around 0.74.

Future Work:

We are going to look at other metrics that describe other types of anisotropic universes. We will see if one of them is a better fit for the supernova data than the isotropic universe, and if the better fit has a large amount of matter in the universe, meaning that dark energy wouldn't be needed to explain the expansion of the Universe.