Quasi-Bosonic Lattice Gas and Information Machines

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Abstract

We used discrete computer models to investigate digital thermodynamic systems. These models have a conserved quantity, which we call energy, so that we were able to define typical thermodynamic quantities. The computer model we used was a descendent of the HPP Lattice gas, but adapted to allow for unbounded energy. We verified the applicability of our model by fitting a Boltzmann Distribution to the microstates of our system.

In Physics we always see microscopic reversibility. In thermodynamics, we do not always see macroscopic reversibility, but we see adiabatic transformations that are *nearly* reversible. Our goal was to implement thermo-dynamic transformations in an exactly reversible way. To do this, we used an information machine that could cause transformations by extracting particles from the gas as bits of information. It would later need to erase its memory by putting particles back into the gas. Our findings showed that microscopic reversibility does not imply macroscopic reversibility. Using this method, there was always a significant entropy and energy increase over a compression and expansion cycle.

Digital Thermodynamics

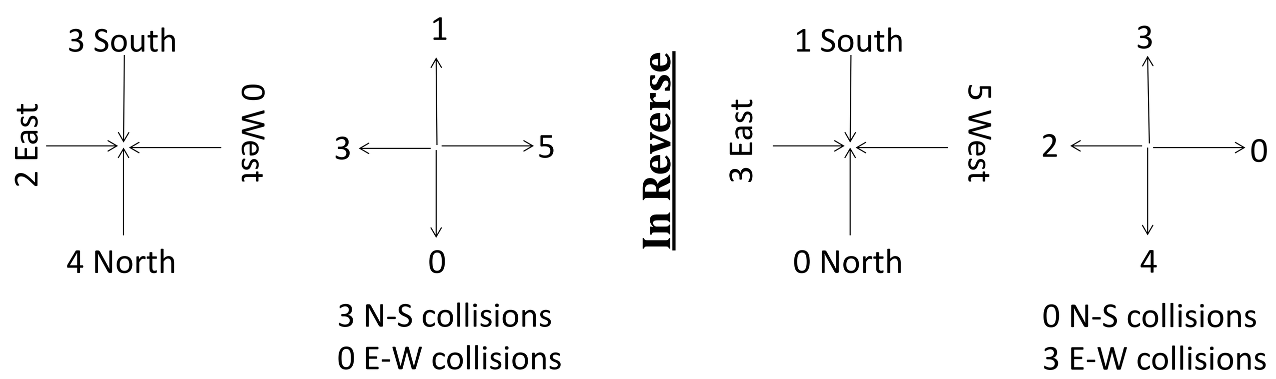
Digital Thermodynamics uses computer models to examine statistical mechanics in a very precise way. All models share 3 characteristics:

1) *They are discrete in time, space, energy, and volume.*
2) *They have a completely reversible time evolution. In other words, they have microscopic reversibility just like everything else we see in Physics.*
3) *There is a conserved quantity, which we take to be energy.*

Quasi-Bosonic Lattice gas (QBL)

In our lattice gas, there are a number of cells that make up a lattice. Each cell in the lattice contains a site for each cardinal direction, and each site has an occupation number corresponding to the number of "particles" at that site. All particles move through the lattice at the same speed, so the "energy" of the gas is the number of particles in the grid at a given time. If no work is done on the gas, energy is always conserved.

The QBL is a descendent of the HPP lattice gas, but in the QBL a site can have occupation numbers greater than 1. This means that there is no maximum energy, whereas an HPP gas cannot have an energy greater than 4 times its volume. The ability to have occupation numbers greater than one is key in compressing and expanding a gas in a microscopically reversible way. Collisions occur when a north-south or east-west pair collide; each colliding particle turns 90 degrees.



Thermodynamic Quantities

When we initialize a QBL gas, we define it to be of size V with an energy equal to the number of particles N. This means that the probability of a given occupation number at a site is given by

$$P(n) = \frac{e^{-\beta n}}{Z}.$$

Where Z is a proportionailty constant given by

$$Z = \sum_{n=0}^{\infty} e^{-\beta n} = \frac{1}{1-e^{-\beta}}$$

We have defined P(n), so we can define entropy for a node, s, by

$$s = \sum_n P(n)\ln\big(P(n)\big) = \sum_n P(n)(\beta n + \ln(Z)) = \beta\langle n\rangle + \ln(Z)$$

By definition, $\langle n\rangle = \sum_n nP(n) = \frac{1}{e^{\beta}-1}$ and we already defined Z.

Also, the entropy of the entire system is given by $S = 4Vs$, so

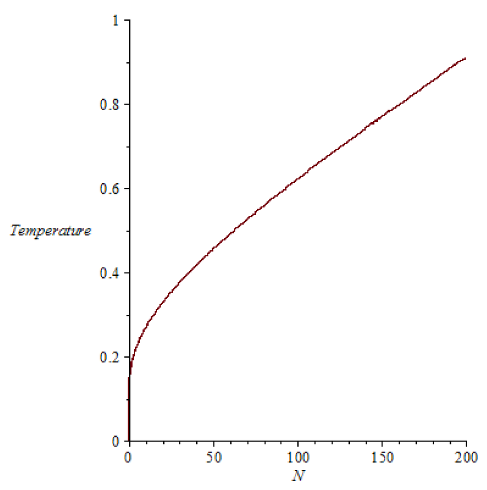
$$S = N\ln(1+\frac{4V}{N}) + 4V\ln(\frac{N}{4V}+1)$$

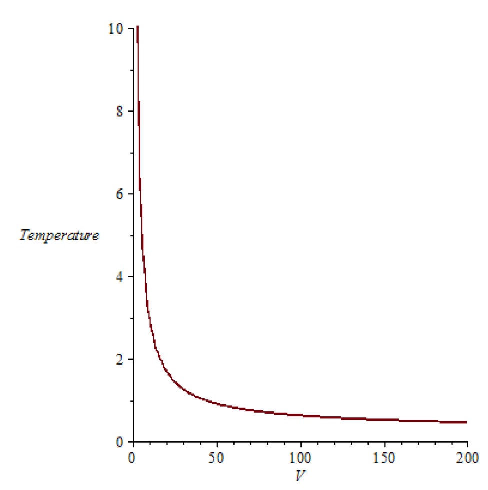
With a definition of entropy, we can define temperature by:

$$\frac{1}{T} = \beta = \left(\frac{\partial S}{\partial N}\right)_V = \ln\left(\frac{4\mathrm{V}}{\mathrm{N}}+1\right)$$

As well as the pressure by:

$$\frac{P}{T} = \left(\frac{\partial S}{\partial V}\right)_N = 4\ln\left(1+\frac{\mathrm{N}}{4\mathrm{V}}\right)$$





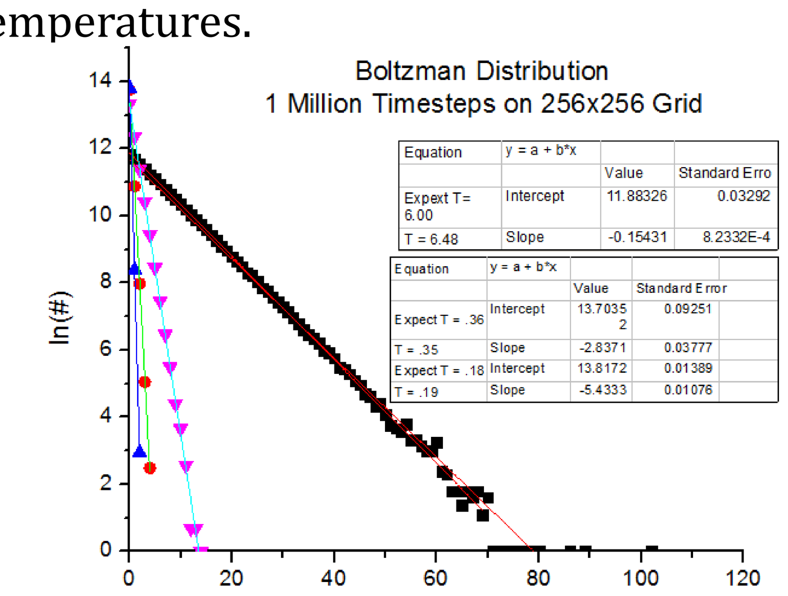
Boltzmann Distribution

For a linearized Boltzmann distribution, we expect

$$P(n)\ \alpha\ e^{-E(n)/T}$$

$$\ln\big(P(n)\big) = C - \frac{1}{T}E(n)$$

Where E(n) is the energy of n. Since our individual particles all have the same energy, we looked at a single occupation site to generate statistics about the probability distribution of energies at various temperatures.



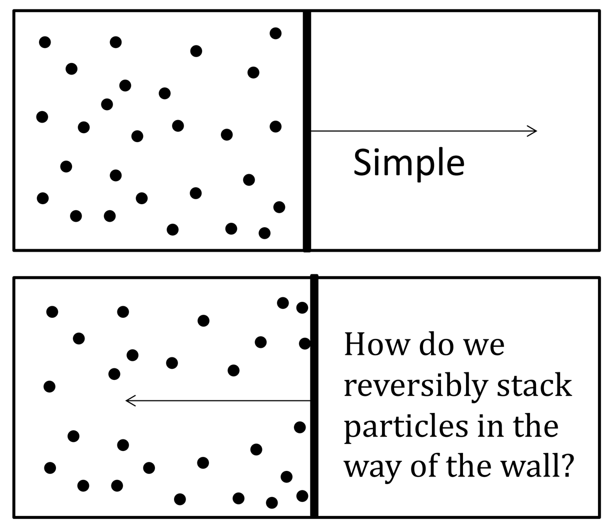
Equation	y = a + b*x		
		Value	Standard Erro
Expext T= 6.00	Intercept	11.88326	0.03292
T = 6.48	Slope	-0.15431	8.2332E-4

Equation	y = a + b*x		
		Value	Standard Error
Expect T = .36	Intercept	13.7035 2	0.09251
T = .35	Slope	-2.8371	0.03777
Expect T = .18	Intercept	13.8172	0.01389
T = .19	Slope	-5.4333	0.01076

We are between a 2% and 8% difference, which indicates that our model is following a Boltzmann distribution. Furthermore, it shows that all of the microstates are equally likely.

Information Machines

One way to define work in our system was to have a wall that can move back and forth to essentially create a piston. Moving the wall so that the volume increases is simple; we can just move the wall and let the particles redistribute through the new volume by free expansion. To move the particles reversibly when the volume is decreasing, we needed to make a one-to-one correspondence between the microstates of the system at high and low volume. Notice that this would be impossible in the HPP gas because the size of the finite set of all microstates depends on the volume. We used an information machine that worked by removing particles in the way of the wall and storing them in a binary list as information. Then the wall would move and the information machine would erase its memory by writing the bits back into the grid. Thus, the one-to-one correspondence we needed for reversibility was just $n_{new} = 2n_{old} + b$ where b is the next bit in the binary list and n is the occupation number of a site. We saw that while this transformation was microscopically reversible it was not macroscopically reversible. Usually the total energy of the system would increase by a significant amount over a compression-expansion cycle. We were also able to show that, on average, information machines like Maxwell's Demon will not violate the second law of Thermodynamics because the cost of information erasure will cause a net increase in entropy.



Prospects

There are many more interesting qualities of information machines left to explore. Since we are compressing and expanding the gas adiabatically, we expect that there would be no net change in entropy if we run a compression and then shortly after run an expansion. Because of the way we are writing particles back into the grid, it is usually the case that the energy, and therefore the entropy, increase considerably. This is true even if we use compression methods on the extracted binary list because for every $4V$ bits that we erase the energy of the system will roughly double. We suspect that there is a more efficient means of information extraction and erasure than the method we are currently using.

References

J. Hardy, Y. Pomeau and O. de Pazzis, "Time evolution of a two-dimensional model system. I. Invariant states and time correlation functions", J. Math. Phys. 14, 1746 (1973).

T. Toffoli and N. H. Margolus, "Invertible cellular automata: A review", Physica D 45, 229 (1990).