Gravitational Perturbations in Lattice Simulations

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Abstract

Lattice simulations are very useful computational cosmology, because they let us test theories and solve differential equations that may not have analytic solutions. We have a very good lattice simulation code: GABE (the Grid And Bubble Evolver), which lets us test cosmological theories on an expanding background. One small drawback to this code is the assumption of a flat space-time, which observation of the universe (and our existence) tells us is not realistic. In response to this, we modified GABE to include Newtonian perturbations to the space-time metric (gravitational perturbations), and tested several theories with this new code.

The Perturbed FLRW Metric

In cosmology, the choice of a spacetime metric is crucial to the outcome of our simulations and experiments. The most common choice is known as the Freidmann-Lemaitre-Robertson-Walker Metric, which is given by

$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & a^2 & 0 & 0 \\ 0 & 0 & a^2 & 0 \\ 0 & 0 & 0 & a^2 \end{pmatrix} \tag{1}$$

This metric depends only time, meaning that it is uniform over all of space. However, evidence (such as gravitational lensing, the existence of galaxies, and our own existence) tell us that in reality, there is a spatial dependence to the metric.

To account for this, we can modify the FLRW Metric to include spatial components.

$$g_{\mu\nu} = \begin{pmatrix} -(1+2\Psi) & 0 & 0 & 0 \\ 0 & a^2(1-2\Phi) & 0 & 0 \\ 0 & 0 & a^2(1-2\Phi) & 0 \\ 0 & 0 & 0 & a^2(1-2\Phi) \end{pmatrix} \tag{2}$$

Where Ψ and Φ are known as the Bardeen variables. Both are dependent on both space and time, so this adds a spatial component to our metric.

Ψ is the Newtonian gravitational potential at a point, and Φ is equal to zero when there is no anisotropic stress present.

Scalar Fields and Lattice Simulations

In computational cosmology, we often find it useful to view the universe as a collection of fields, because they are much easier to simulate than particles. We have a program called GABE: the Grid And Bubble Evolver, which evolves the equations of motion of scalar fields with the second order Runge-Kutta method. We calculate the value of the fields at specific points, in a three dimensional grid. This is known as a Lattice Simulation.

Perturbed Equations of Motion

We take the base code of GABE and modify it to include the gravitational perturbation terms, which in turn feed into the equations of motion for the other fields that we are simulating. Normally, the equation of motion of a scalar field is given by the Klein-Gordon Equation

$$\ddot{\phi} = \frac{\nabla^2\phi}{a^2} - \frac{\partial V}{\partial \phi} - 3\frac{\dot{a}}{a}\dot{\phi} \tag{3}$$

When we make our metric the Perturbed FLRW metric, then we get a variation of the Klein-Gordon Equation, which we then can put into GABE to evolve our fields.

$$\ddot{\phi} = 2\frac{\dot{\Psi}\dot{\phi}}{1+2\Psi} + 2\delta_i^j\frac{\partial_i\phi\partial_j\Phi}{a^2(1-4\Phi)} + \frac{\nabla^2\phi(1+2\Psi)}{a^2(1-4\Phi)} - (1+2\Psi)\frac{\partial V}{\partial\phi} - 3\frac{\dot{a}}{a}\dot{\phi} \tag{4}$$

Next, we solve the Einstein Equations for the equations of motion for the perturbation fields. These end up being extremely complicated, which makes them an especially good candidate for numerical simulation.

$$\ddot{\Phi} = \frac{1}{3a^2}(\nabla^2\Phi - \nabla^2\Psi) - \frac{\dot{a}}{a}(3\dot{\Phi} + \dot{\Psi}) + 4\pi G\Big(\big((1+2a^4)\Phi + (1+2a^2-2a^4)\Psi\big)\delta_i^j\partial_i\phi\partial_j\phi + \big(\Phi + (2-2a^2)\Psi\big)V(\phi)\Big) \tag{5}$$

$$\dot{\Psi} = \frac{1}{3a\dot{a}}(\nabla^2\Phi - \nabla^2\Psi) - 3\dot{\Phi} + 4\pi G\Big(\big((1+2a^4)\Phi + (1+2a^2-2a^4)\Psi\big)\delta_i^j\partial_i\phi\partial_j\phi + \big(\Phi + (2-2a^2)\Psi\big)V(\phi)\Big) - \frac{a}{\dot{a}}\ddot{\Phi} \tag{6}$$

Preheating

The Standard Model of Cosmology has several issues, including the "Horizon Problem", which becomes apparent when we look at the Cosmic Microwave Background. It is almost uniform, which doesn't make sense, as opposite sides should not be causally connected.

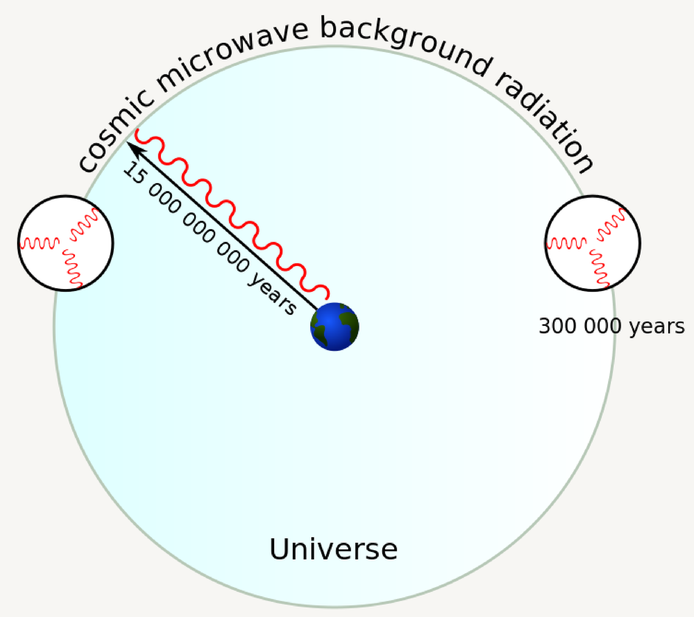

Figure: The Horizon Problem [**Knott:2010**]

In the 1980s, Alan Guth pioneered the theory of Cosmic Inflation, which was a period of exponential expansion of the universe just after the Big Bang. This period saw an increase of the scale factor of the universe by about a factor of at least 10^{26}.

This solves the horizon problem, but introduces another: at the end of inflation, the universe has nothing familiar to us in it: no matter, no electromagnetic waves.

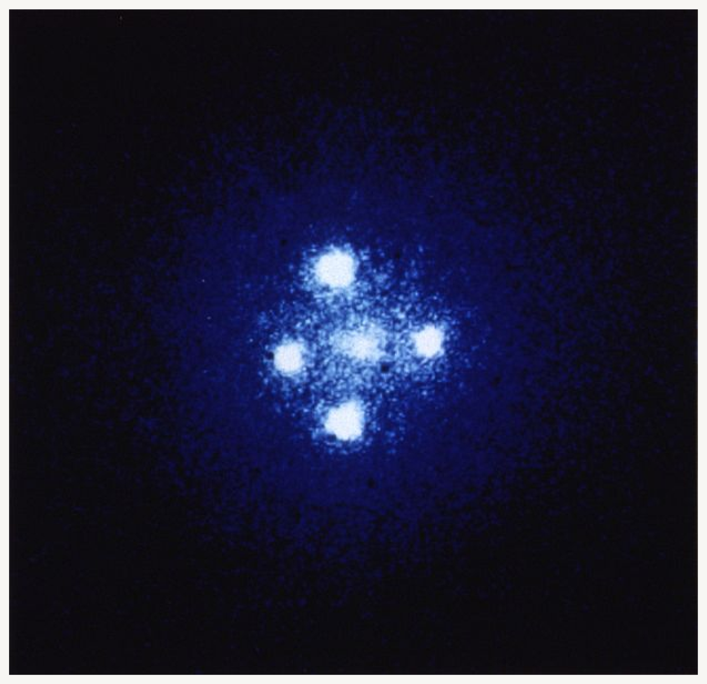

Figure: Evidence of Stuff in Universe [**NASA:1990**]

In response to this, the theory of preheating was introduced. This period was characterized by a large increase in variation in the inflaton field, which was coupled to a matter field. In this way, the energy from the inflaton field was transferred to reservoirs of energy that we would find familiar.

Testing this model in the modified GABE code, we found that it was consistent with the non-perturbed simulations that we had been running. Although we didn't find anything new, this is encouraging, because it means that our old simulations were valid!

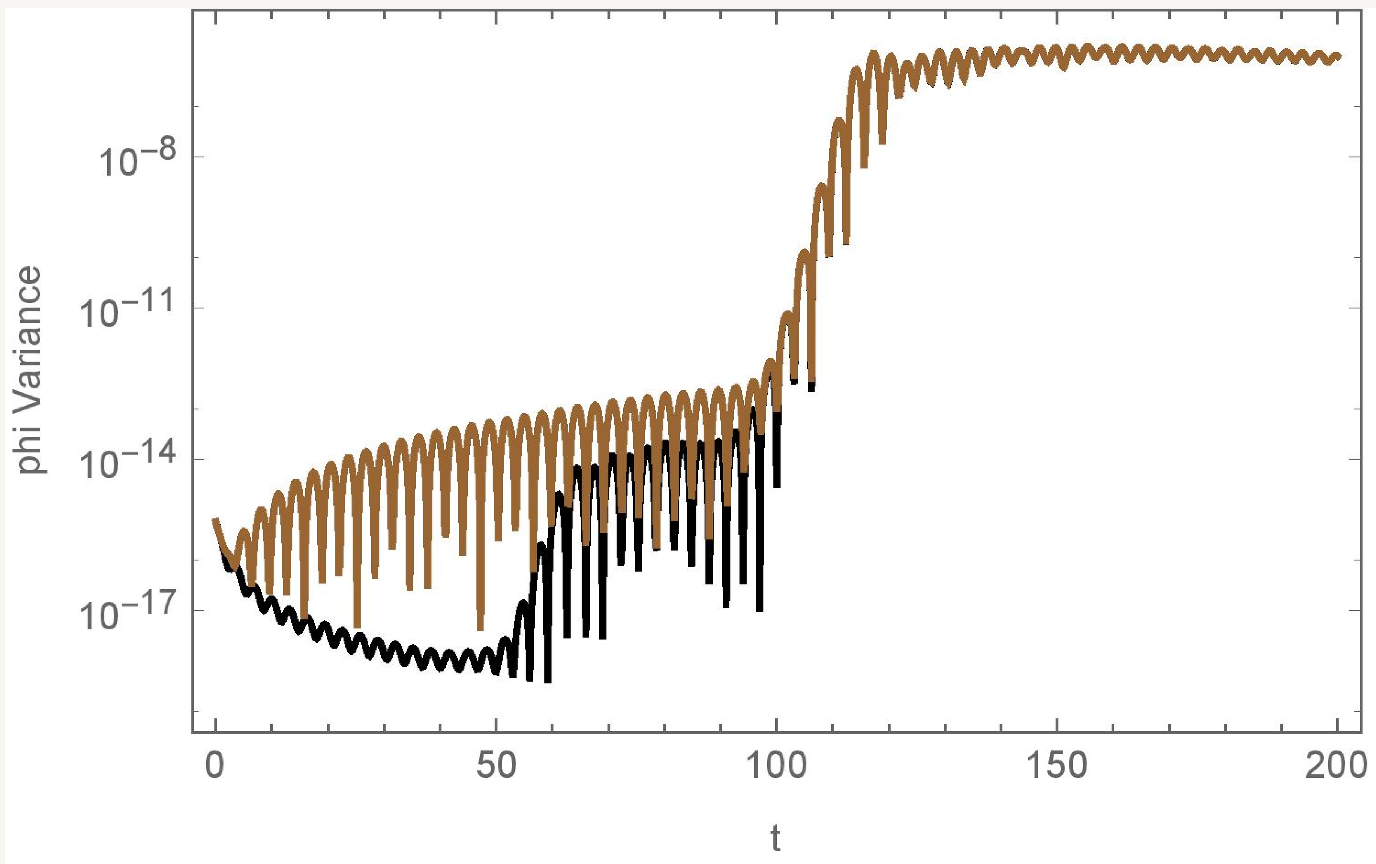

Figure: Evidence of Stuff in Universe [**NASA:1990**]

Future Work

Now that we have a fully working code, we are working towards more new and exciting applications for our code. We have talked to researchers at MIT and Tufts about using our code to test theories involving scalar field dark matter as a method for the formation of structure in the early universe.

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