# Dynamical Studies of Non-Abelian Gauge Fields: Cosmic Inflation and Reheating

Peter Adshead[1] John T. Giblin, Jr[2] Zach Weiner[2]

[1]Dept. Of Physics, University of Illinois at Urbana-Champaign

[2]Dept. Of Physics, Kenyon College

Kenyon College Physics

## Abstract

Introduced by Ken Wilson in 1974, lattice gauge theories have emerged as a useful tool for numerical simulations of gauge fields. Lattice approximations of actions and equations of motion involving gauge fields are a natural choice for dynamical simulations of four-dimensional spacetime, which must already be discretized to a lattice to be numerically tenable. In particular, we can apply these methods to study reheating by evolving the inflaton coupled to various Standard Model particles. Classical simulations in the $U(1)$ case have been successful; we now turn to the more challenging non-abelian case. The viability of classical evolution of $SU(2)$ gauge fields has been contested, calling for lattice implementations. However, the higher resolution offered by current technology offers a chance to successfully evolve the classical theories, which we will subsequently verify with the lattice theory. In particular, we will evolve the inflaton coupled to an $SU(2)$ field via a dilatonic coupling.

## Reheating

Inflation, while resolving three major problems of early-universe Cosmology, leaves the universe cold. All of the energy that goes into creating matter, radiation, etc., as we see in the present universe is trapped inside the inflaton field. Reheating is the process through which the Universe thermalizes, converting the inflaton energy into radiation which eventually precipitates matter. Preheating, a particularly efficient model of reheating, achieves thermalization by releasing the trapped energy in the inflaton as it oscillates via non-linear processes such as parametric resonance, similar to a child pumping their legs to swing higher. We aim to simulate inflaton-gauge field couplings to see if parametric resonance occurs.

## Gauge Theory

The defining property of a gauge theory is an action which is invariant under symmetric transformations. Gauge fields arising from such theories are force mediators in the Standard Model. The simplest example, the $U(1)$ abelian group, describes QED; we call its gauge field the photon.

We consider the Special Unitary Group, $n = 2$, denoted by $SU(2)$. Its elements are $2 \times 2$ unitary matrices with determinant 1, generated by the Pauli matrices,

$$\sigma^a = \left[ \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right]. \tag{1}$$

$SU(2)$ fields have three "flavors," one associated with each generator. The $SU(2)$ field tensor is

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + i g_W \epsilon^{abc} A^b_\mu A^c_\nu, \tag{2}$$

where $A^a_\mu(x)$ is the vector potential of the gauge field. The third term in the field tensor arises from the non-abelian nature of $SU(2)$.

The Yang-Mills action, the typical gauge-invariant choice, is

$$S_{YM} = -\frac{1}{4} \int d^4x \sqrt{-g}\, \mathrm{Tr} \left[ F^a_{\mu\nu}(x) \sigma^a F^{a,\mu\nu}(x) \sigma^a \right]. \tag{3}$$

## The Continuum Model

Our model explores the interaction between the inflaton $\phi$ and an $SU(2)$ gauge field under the dilatonic coupling $\Theta(\phi(x)) = \exp(\phi/M)$. Using the shorthand $\mathcal{S}_G = -\frac{1}{4}\mathrm{Tr}\left[F^a_{\mu\nu}(x)\sigma^a F^{a,\mu\nu}(x)\sigma^a\right]$, our action is

$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} \partial_\mu \phi(x) \partial^\mu \phi(x) + V(\phi(x)) + \Theta(\phi(x)) \mathcal{S}_G \right]. \tag{4}$$

We will study both continuum and lattice forms of the equations of motion. The continuum equations of motion are [3]

$$\Box \phi(x) + V'(\phi(x)) + \Theta'(\phi(x)) \mathcal{S}_G(x) = 0 \tag{5}$$

and

$$\Box A^a_\mu(x) + \frac{W'(\phi(x))}{W(\phi(x))} \eta^{\nu\beta} F^a_{\mu\nu}(x) \partial_\beta \phi(x) = 0. \tag{6}$$

## The Lattice Approximation

To discretize a gauge theory, we approximate spacetime as a four-dimensional lattice with spatial spacing $a$ and temporal spacing $a_t$. Our gauge field becomes a field of *link variables*, defined by

$$U_\mu(x) = \exp\left[ -\frac{i a_\mu g_W}{2} \sigma^a A^a_\mu(x) \right]. \tag{7}$$

The variable $U_\mu(x)$ lives on the "link" between the lattice point at $x$ and the adjacent lattice point in the $\mu$-direction. We also define the *plaquette*

$$P_{\mu\nu}(x) = U_\mu(x) U_\nu(x + \hat{\mu}) U^\dagger_\mu(x + \hat{\mu}) U^\dagger_\nu(x). \tag{8}$$

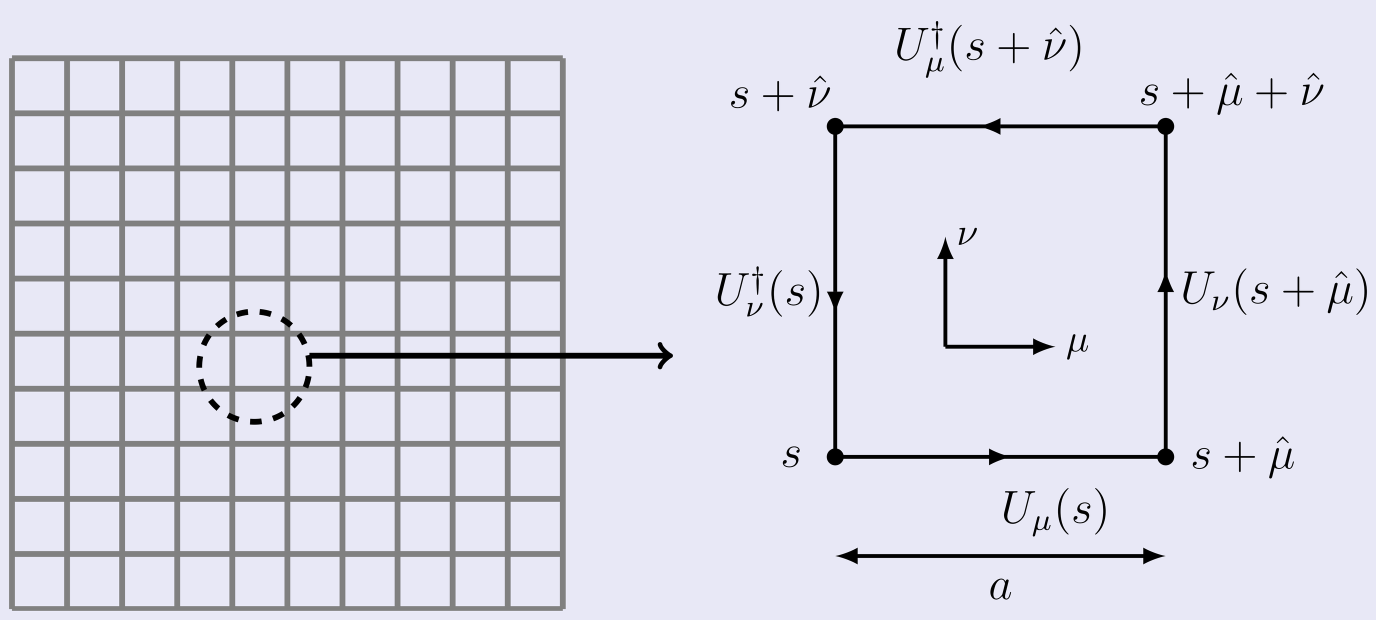


Figure: The plaquette $P_{\mu\nu}(s)$. [2]

The plaquette is particularly useful as an approximation for the field tensor; expanding in order of the lattice spacing results in

$$P_{\mu\nu}(x) = 1 - \frac{i a_\mu a_\nu g_W}{2} F^a_{\mu\nu}(x) - \frac{a^2_\mu a^2_\nu g^2_W}{8} \left( F^a_{\mu\nu}(x) \right)^2 + \cdots. \tag{9}$$

By antisymmetry, the first order term vanishes when approximating the Yang-Mills action:

$$S_L = -\frac{1}{4} \sum_x \kappa a^4 \frac{2}{g^2} \sum_{\mu<\nu} \Re \mathrm{Tr}\left[1 - P_{\mu\nu}(x)\right]. \tag{10}$$

Fixing a temporal gauge results in $U_0(x) = 1$.

## Lattice Equations of Motion

Variation on the lattice action with respect to $\phi$ gives the same equation of motion as in the continuum:

$$\Box_L \phi(x) + V'(\phi(x)) + \Theta'(\phi(x)) \mathcal{S}_G(x) = 0. \tag{11}$$

$\Box_L$ denotes the lattice D'alembertian, which differs only by the metric used. Varying with respect to the links, we find that

$$\sum_{\mu \neq \gamma} \Theta(\phi(x)) W_{\gamma,\mu}(x) + \sum_{\mu \neq \gamma} \Theta(\phi(x - \hat{\mu})) W_{\gamma,-\mu}(x) = 0. \tag{12}$$

The term $W_{\gamma,\mu}(x)$ indicates a *staple*, which is defined such that $U_\gamma(x) W_{\gamma,\mu}(x) = P_{\gamma\mu}(x)$. To decouple this second order equation (staples resolve roughly to second order derivatives), we introduce the electric field

$$E^a_i(x) \sigma^a = U_i(x + a_t) U^\dagger_i(x) - 1 \tag{13}$$

which allows us to define an "update rule" for the links via

$$U^a_i(x + a_t) = E^b_i(x) U^c_i(x) \epsilon^{abc} + U^a_i(x). \tag{14}$$

Then, substituting into Equation (12),

$$\begin{aligned} E^a_i(x) = &-\frac{\Theta(\phi(x - \hat{0}))}{\Theta(\phi(x))} E^a_i(x - \hat{0}) + \frac{\kappa^2}{2} \sum_{j \neq i} P^a_{ij}(x) \\ &+ \frac{\kappa^2}{2} \sum_{j \neq i} \frac{\Theta(\phi(x - \hat{\jmath}))}{\Theta(\phi(x))} P^a_{ij}(x - \hat{\jmath}). \end{aligned} \tag{15}$$

This closes our solution set.

## Future Work

A version of GABE [1] modified to evolve gauge fields in accordance with lattice gauge theories is being finalized. We cannot use GABE's native second-order Runge-Kutta method, since we do not consider points in between lattice sites to exist. This leaves Euler's method as the only viable option. We will evolve the dilatonic coupling with continuum equations of motion as well in order to compare results at resolutions upwards of $512^3$. The results of simulations will allow us to both evaluate the viability of lattice approximations as well as $SU(2)$ coupling as a reheating model.

### *Acknowledgements and References*

Thanks to Professor Tom Giblin, the Kenyon Summer Science Scholars Program, and the Kenyon College Department of Physics. This project is supported by NSF Grant 1414479.

[1] URL: http://cosmo.kenyon.edu/gabe.html.

[2] Nuno Cardoso and Pedro Bicudo. "SU(2) Lattice Gauge Theory Simulations on Fermi GPUs". J.Comput.Phys. 230 (2011), pp. 3998–4010. arXiv: 1010.4834 [hep-lat].

[3] J. Tate Deskins, John T. Giblin, and Robert R. Caldwell. "Gauge Field Preheating at the End of Inflation". Phys.Rev. D88.6 (2013), p. 063530. arXiv: 1305.7226 [astro-ph.CO].

Dynamical Studies of Non-Abelian Gauge Fields: Cosmic Inflation and Reheating