Recovering the Neutron Star Equation of State from a Binary Neutron Star Gravitational Wave Detection with LIGO

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Abstract

During the collision of two orbiting neutron stars, the gravitational gradient across one due to the other causes tidal deformations in the neutron star. This deformation causes a change in quadrupole moment of the binary system, which in turn alters the gravitational waveform emitted during the inspiral and eventual collision of the two neutron stars. Such an alteration of the gravitational waveform can give us insight into the behavior and structure of neutron star matter in the form of constraints on the neutron-star equation of state. We can generate waveforms that include alterations due to tidal deformations and use parameter estimation (PE) to measure the model parameters and constrain the equation of state (EOS) in general. For this project, we incorporated an EOS model into LIGO's fully Bayesian PE routines and tested the effectiveness of this method of EOS measurement.

Background

Currently, few constraints have been imposed on the neutron-star EOS. Electromagnetic observations of neutron stars have given accurate measurements of neutron-star (NS) masses, however measurements of radii from optic observations are much more challenging to make. The recent gravitational-wave detection made by LIGO introduces a new type of astronomy useful for constraining the neutron-star EOS. Anticipating an NS GW detection, we simulate a binary neutron star gravitational-wave detection and test one EOS parameterization’s recoverability using LIGO’s Bayesian parameter estimation routine.

Approach

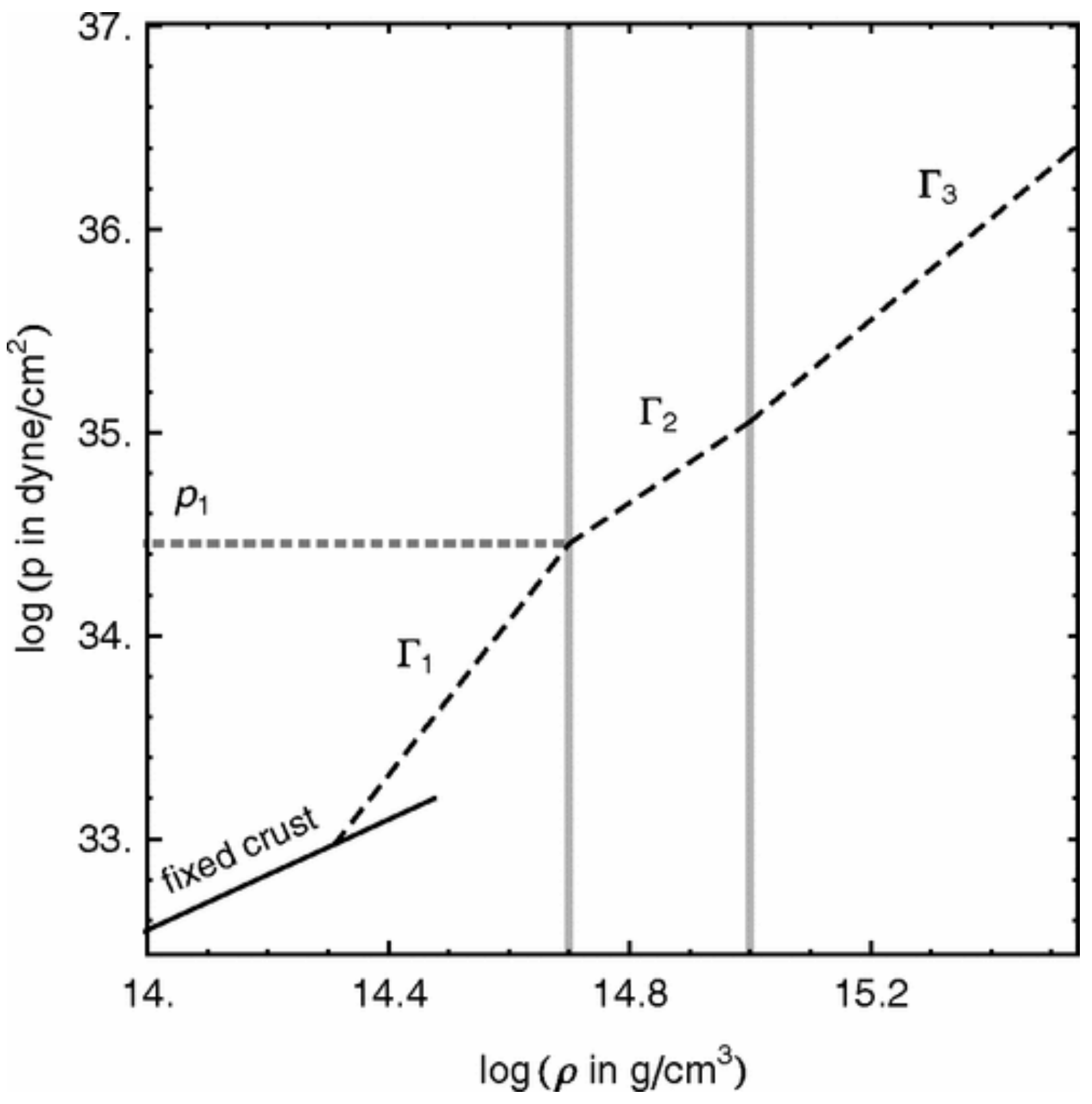

Fig 1: 4-piece polytrope EOS.

One parameterization of the neutron-star equation of state is the 4-piece polytrope. The three adiabatic indices, denoted by Γ, which indicate the slope of the corresponding segment, along with p_1, which specifies an anchor point to the low-density fixed-crust EOS, specifies a neutron-star EOS completely.

The discontinuities in this parameterization represent phase shifts in the neutron-star matter. Other parameterizations of the neutron-star EOS include the one-parameter polytrope and a spectral decomposition of the adiabatic index, the latter of which Professor Wade and I hope to explore in the future.

Neutron Star Equation of State

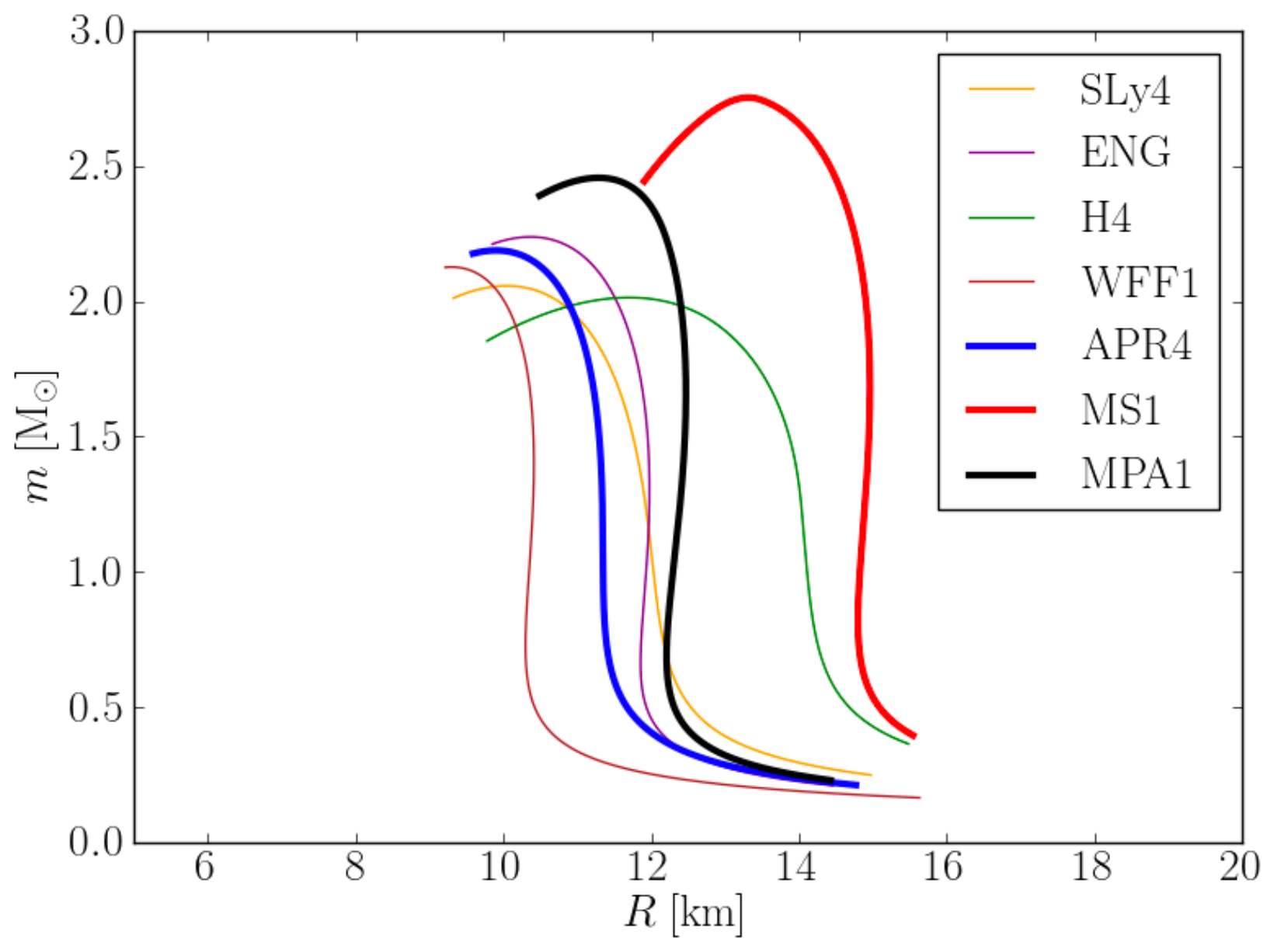

Fig 2: Various theorized EOS models.

In thermodynamics, an equation of state relates the pressure *p* of a system to its density ρ. In the case of the neutron-star equation of state, solving the Tolman-Oppenheimer-Volkoff (TOV) equations yields a one-to-one mapping from the pressure-density space p(ρ) to the mass-radius space M(R)[3]. In other words, for a given mass, the neutron-star EOS tells us how much a neutron star’s radius will decrease if a certain pressure is applied uniformly to its surface.

Parameter Estimation

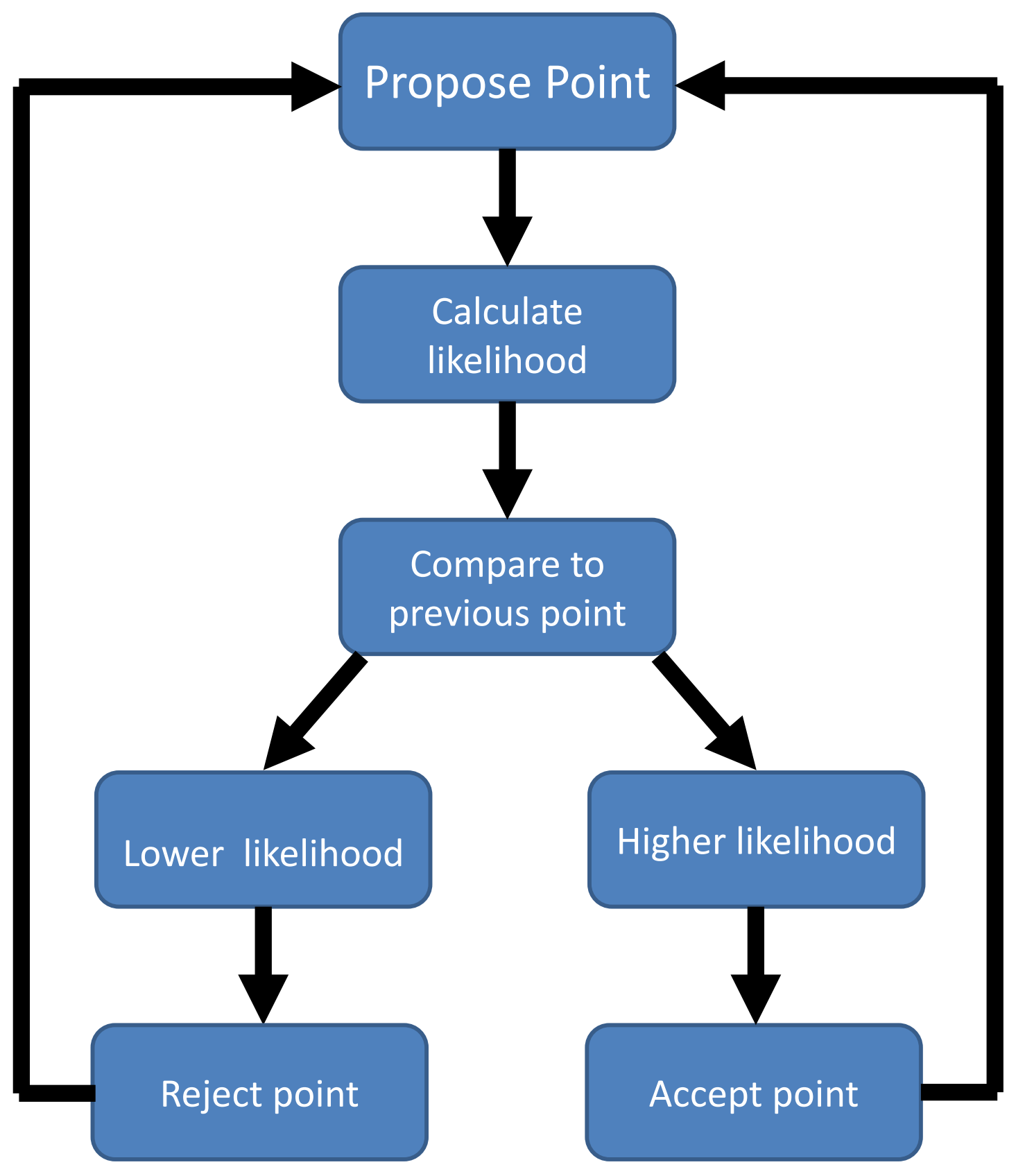
Propose Point
Calculate likelihood
Compare to previous point
Lower likelihood
Higher likelihood
Reject point
Accept point

Fig 3: Bayesian parameter estimation pipeline.

LIGO uses parameter estimation to construct probability densities for a 16-dimensional parameter space. These parameters include those characteristic of the physical configuration of the system, such as the masses and spins of the two neutron stars, as well as parameters that arise due to the relative locations of the detectors and the source of the signal, such as the sky location of the system. We used a Markov chain Monte Carlo method in conjunction with a Bayesian statistical approach to determine likelihoods for each proposed set of parameters. We include prior information about the EOS to make smarter parameter proposals. One of the most useful, but perhaps most obvious, is the causality constraint: The proposed EOS must not allow for a speed of sound within the neutron star greater than the speed of light. We can further constrain our parameter proposals by eliminating EOS that don’t allow for neutron stars with masses or radii already known to exist through other observations.

Conclusions

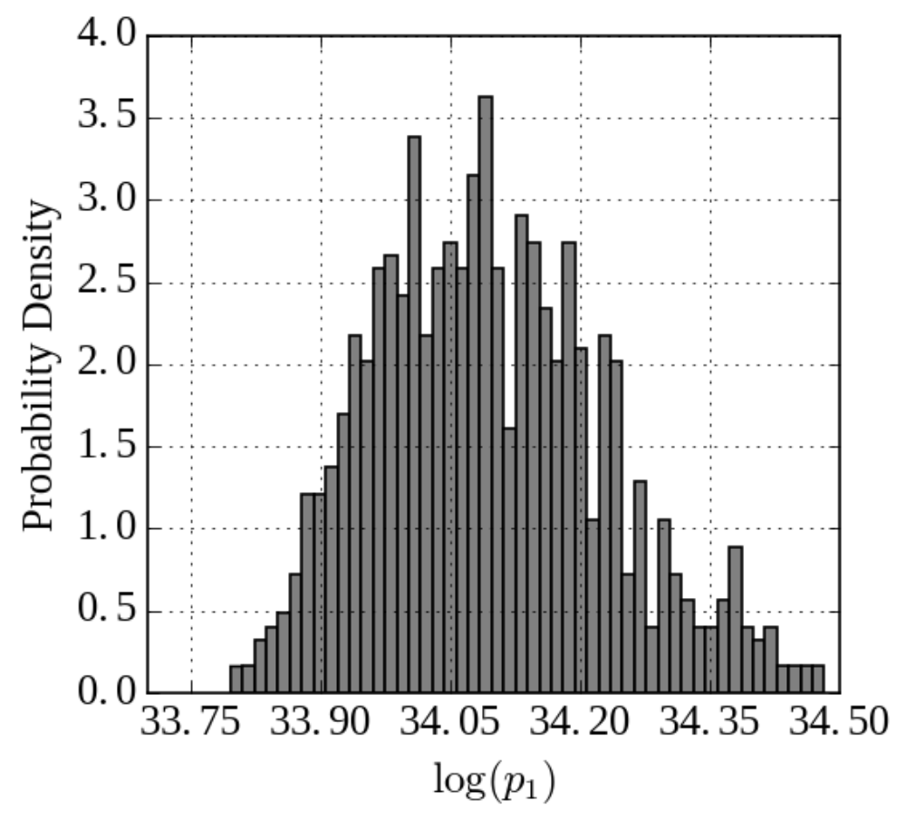

Fig 4. Probability density distribution for the EOS parameter *log(*p_1*)*.

While we made significant progress this summer incorporating the algorithm, we’re still testing it through various simulations and equations of state.

Overall, we found that *log(*p_1*)* is the most constrained in this parameterization, while Γ_1, Γ_2, and Γ_3 were relatively unconstrained. Even constraining one parameter would be a very useful tool in constraining the entire EOS. This would correspond to knowing *part* of the neutron-star equation of state, while the rest was essentially unconstrained.

Future Work

Professor Wade and I are currently working on implementing a spectral-decomposition representation of the neutron-star EOS into the LIGO Algorithm Library for use in parameter estimation of future binary neutron star gravitational wave detections. Previous publications have showed promising results for the spectral decomposition’s ability to recreate EOS of all types, even those with discontinuities due to phase shifts in neutron star matter.

References

[1] Wade IV, Leslie, “Gravitational Waves from Rotating Neutron Stars and Compact Binary Systems” (2015). Theses and Dissertations.

[2] Lackey, Benjamin, “Reconstructing the neutron-star equation of state with gravitational-wave detectors from a realistic population of inspiralling binary neutron stars” (2015). Phys. Rev. D 91, 043002.

[3] Lindblom, Lee, “Spectral Representations of Neutron-Star Equations of State” (2010). Phys. Rev. D 82, 103011.

[4] Read, Jocelyn S., Lackey, Benjamin, et al., “Constraints on a phenomenologically parameterized neutron-star equation of state” (2009). Phys. Rev. D 79, 124032

Acknowledgments

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