Reversible Ising Dynamics: a Digital Thermodynamic Model

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Abstract

We investigated the behavior of the reversible Ising dynamics system—a simple and explicitly reversible model of the microscopic dynamics of magnetic systems. We searched for ways to quantify the system's thermodynamic behavior, including a probe for the local temperature of the system using Boltzmann statistics of a localized coupled two level subsystem. We considered a novel extension to the Reversible Ising Dynamics Model where we alter the system's Hamiltonian to model work performed or extracted via the changes in an external field. We further investigate the reversibility of such magnetization and demagnetization processes using a fixed work schedule and investigated the consequences of using the sparsely or gradually applied fields. We ultimately used the coupled subsystem approach to integrate a bit engine model of Maxwell's Demon (a system that extracts work at the cost of recording information about our system).

Introduction

The **Ising model** approximates magnetic systems and their transition between paramagnetic and ferromagnetic phases.[1] For this project, we use a scheme to model the microscopic dynamics of Ising systems. The internal dynamics of the system are explicitly reversible and conserve the total energy of the system (defined as the familiar Ising energy). While the dynamics of the system are cartoonishly simply, they capture the essence of thermodynamic behavior.

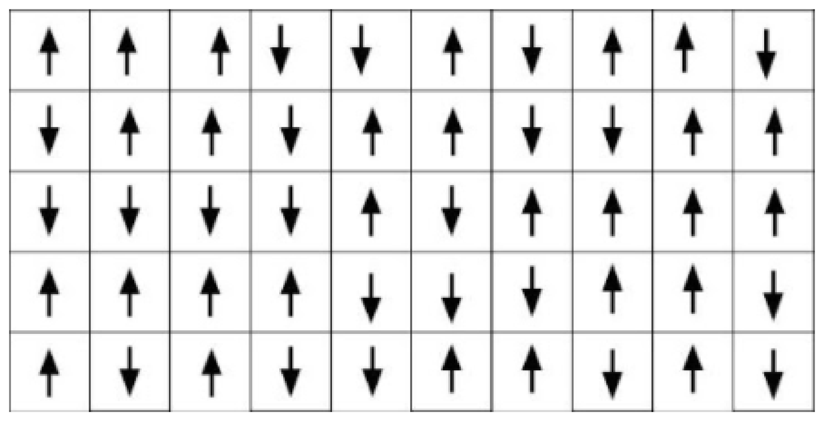

Figure 1: A 2D Lattice of Spins

Reversible Ising Dynamics

Our model consists of a finite two dimensional grid of spins, whose energy is determine by the number of stressed (anti-aligned) or relaxed (aligned) nearest neighbor bonds. The system energy is then[1]:

$$E = -\sum_{i,j} s_i s_j \quad (1)$$

We model the microscopic behavior over a series of discrete time steps (i.e. the time can only take values of 0,1,2 etc.) according to the following local rule:

- System energy must always be conserved
- Whenever flipping a spin would conserve energy, flip the spin.

To ensure global invertibility, (corresponding to the time reversibility in physics) we use a partitioning scheme.[2][3]

Exact Reversibility

- We partition the grid of spins into two subgrids like the black and white squares on a chessboard
- Cells update on alternating substeps; if a cell updates on a substep, none of its neighbors must update on the same substep
- Cells corresponding to black and white squares update on different substeps

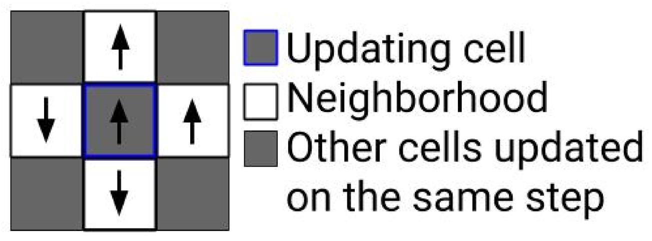

Figure 2: A spin and its neighborhood

Introducing External Fields

One goal for this project was to extend Reversible Ising dynamics to situations involving thermodynamic work (previous work has already examined heat flow in Ising systems[4]) The new energy is:

$$E = -\sum_{i,j} s_i s_j - \sum_i B_i s_i \quad (2)$$

The change in energy due to a change in the field is the work performed on the system (this can be positive or negative corresponding to adding or extracting energy from the system.)

For the Ising system in a paramagnetic phase, we can see the net magnetization change with an applied field. We apply the field 'sparsely' at every third cell in every third row and 'gradually' by waiting several time steps in between magnetizations.

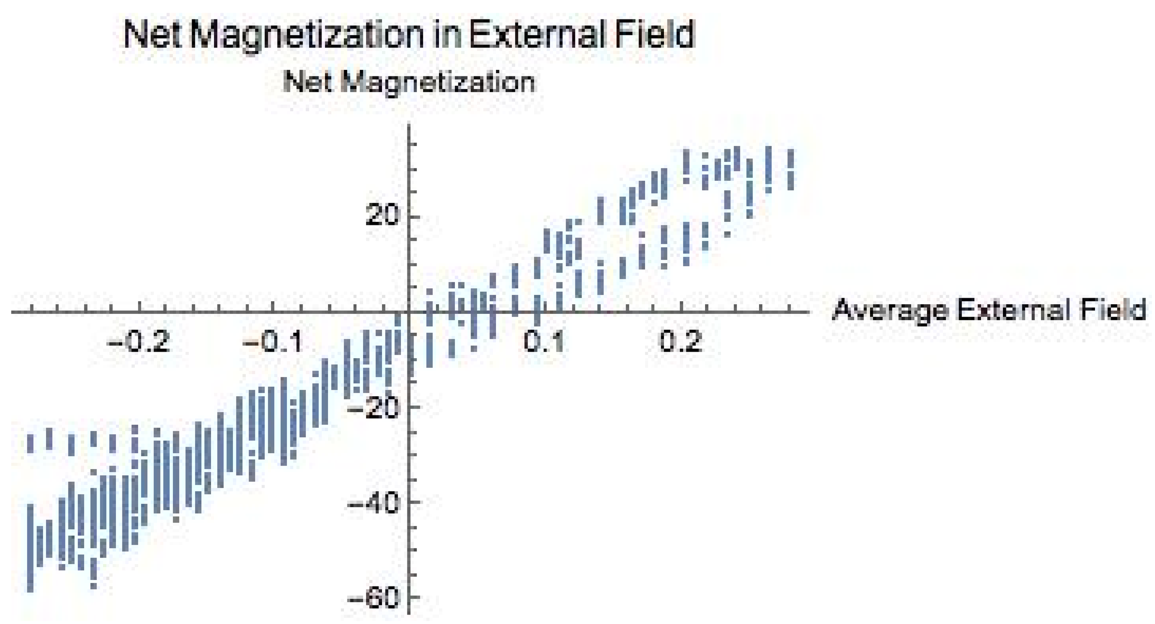

Figure 3: The net magnetization of a grid changes with the applied field

Measuring Local Temperature

We used a Metropolis algorithm to generate initial conditions for our grid of spins at a particular temperature. This corresponds to a global initial temperature, but we want to characterize the temperature of a specific location in our grid, so we can see how the temperature changes as we subject our system to various processes. Based on the Boltzmann distribution, we find the inverse temperature β to be related to how often the subsystem is found in its two states:

$$\frac{p(1)}{p(0)} = e^{-\beta \Delta E} \implies \beta = \Delta E \ln \frac{p(1)}{p(0)} \quad (3)$$

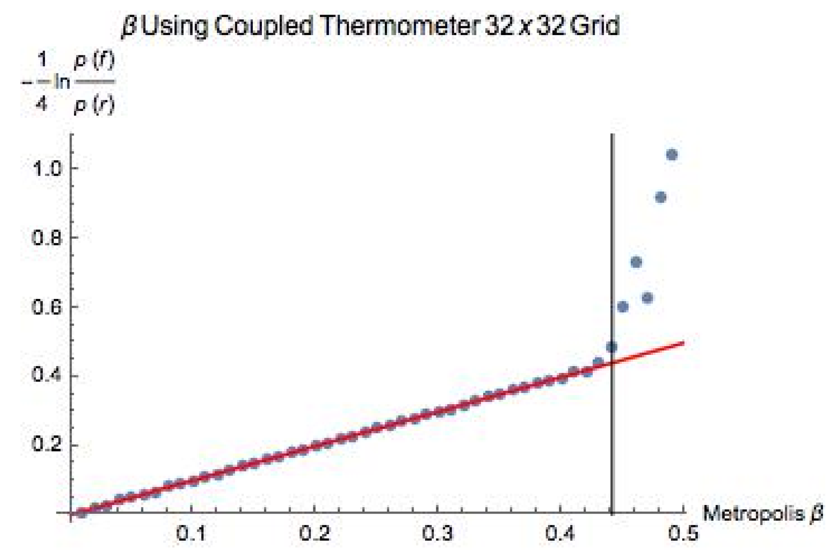

Figure 4: Comparing measured temperature to set temperature

Maxwell's Demon

Maxwell's Demon is a theoretical device that measure a system's microstate and uses that information to extract work. To model this process concretely, we consider a finite state demon that can manipulate 'bits' coupled to our environment as well bits in a memory tape. The demon interacts with two state 'bits' coupled to our Ising grid at specific points. This integrated the work of Avery Tishue on reversible bit engines.

Conclusion

We extended the Reversible Ising Dynamics model with a coupled substem acting as a 'thermometer' to measure the **local temperature**. We also introduced a model for **work done on the system**, changing the system dynamics corresponding to a changed external field. We investigated modeling **cyclic processes** in our discrete world (corresponding to thermodynamics cycles). Finally, we used our two state subsystems to act as thermal bits in a simple model of **Maxwell's Demon**—an explicitly reversible model of the Demon interacting with a thermal environment

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