Recovering the Neutron Star Equation of State from a Simulated Binary Neutron Star Gravitational-wave Detection with LIGO

Matthew Carney ’18, Burke Irwin ’19 and Leslie Wade.
Department of Physics, Kenyon College, 2017

Abstract

During the collision of two orbiting neutron stars, the gravitational gradient across one due to the other causes tidal deformations in the neutron star. This deformation causes a change in quadrupole moment of the binary system, which in turn alters the gravitational waveform emitted during the inspiral and eventual collision of the two neutron stars. Such an alteration of the gravitational waveform can give us insight into the behavior and structure of neutron star matter in the form of constraints on the neutron-star equation of state. We can generate waveforms that include alterations due to tidal deformations and use parameter estimation (PE) to measure the model parameters and constrain the equation of state (EOS) in general. For this project, we incorporated an EOS model into LIGO's fully Bayesian PE routines and tested the effectiveness of this method of EOS measurement. This method performed well, but had increased systematic error due to certain regions intrinsic to the model. This motivated a new model which was the second half to my project this summer.

Background

Currently, few constraints have been imposed on the neutron-star EOS. Electromagnetic observations of neutron stars have given accurate measurements of neutron-star (NS) masses, however measurements of radii from optic observations are much more challenging to make. The recent gravitational-wave detection made by LIGO introduces a new type of astronomy useful for constraining the neutron-star EOS. Anticipating an NS GW detection, we simulate a binary neutron star gravitational-wave detection and test one EOS parameterization’s recoverability using LIGO’s Bayesian parameter estimation routine.

Preliminary Approach

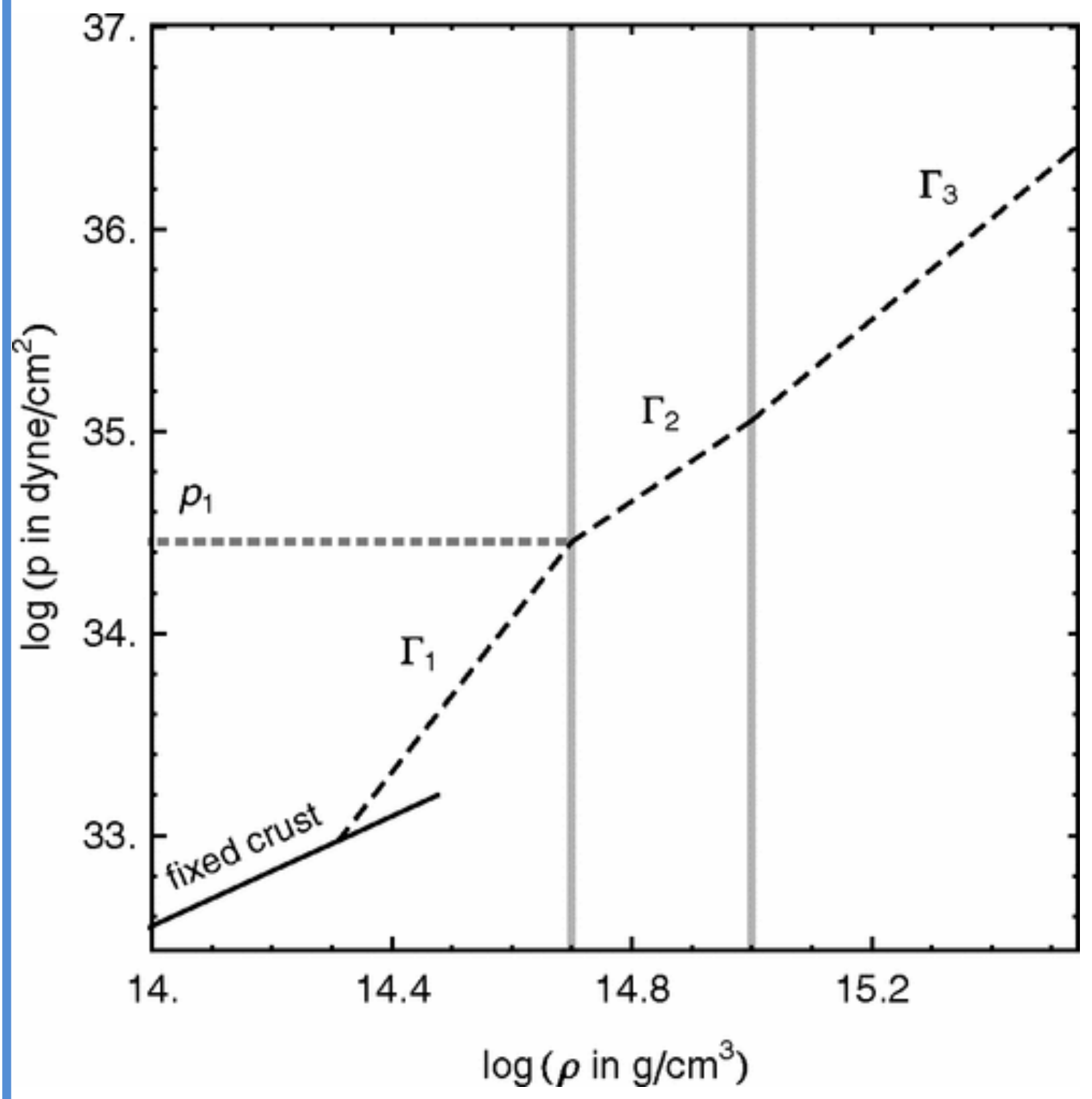

Fig 1: 4-piece polytrope EOS [4].

One parameterization of the neutron-star equation of state is the 4-piece polytrope. The three adiabatic indices, denoted by Γ, which indicate the slope of the corresponding segment, along with p_1, which specifies an anchor point to the low-density fixed-crust EOS, specifies a neutron-star EOS completely.

The 4-piece polytrope model has been shown to fit well to myriad candidate equations of state. Moreover, fits to each of those candidates reproduce macroscopic observables with few percent error [4].

Neutron Star Equation of State

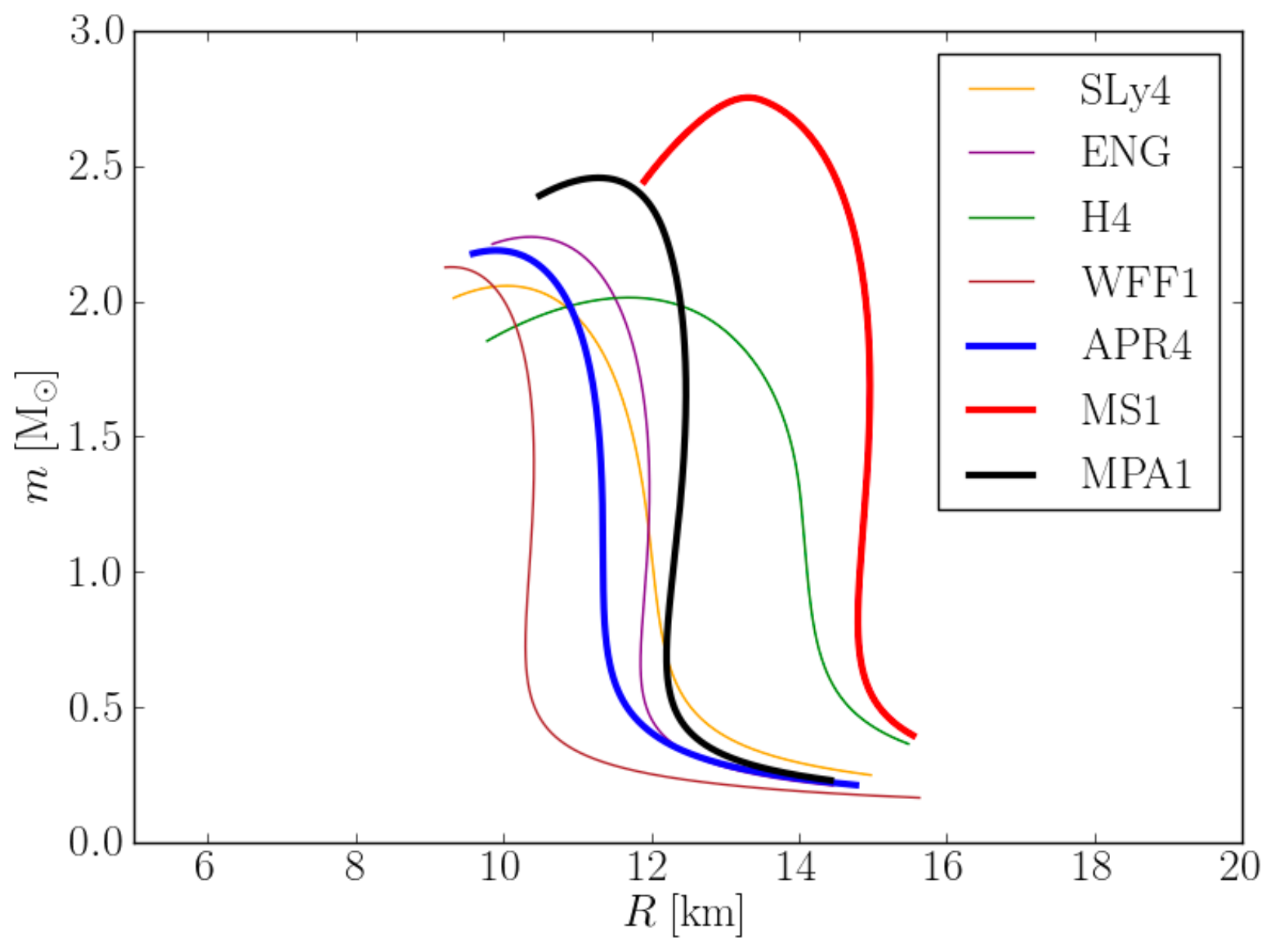

Fig 2: Various theorized EOS models.

In thermodynamics, an equation of state is a relationship between state variables, such as pressure and density. In the case of the neutron-star equation of state, solving the Tolman-Oppenheimer-Volkoff (TOV) equations yields a one-to-one mapping from the pressure-density space p(ρ) to the mass-radius space M(R)[3]. In other words, for a given mass, the neutron-star EOS has just one possible radius value.

Follow-up Approach

$$\Gamma(x) = \exp\left(\sum_k \gamma_k x^k\right)$$

where $x = \log(p/p_0)$

$$\epsilon(p) = \frac{\epsilon_0}{\mu(p)} + \frac{1}{\mu(p)}\int_{p_0}^{p} \frac{\mu(p')}{\Gamma(p')}dp'$$

where $\mu(p) = \exp\left[-\int_{p_0}^{p} \frac{dp'}{p'\Gamma(p')}\right]$

Fig 3: Spectral decomposition of adiabatic index [3].

While the 4-piece polytrope fit to the EOS reproduced macroscopic observables well, the parameter estimation techniques used to recover fit parameters yielded large systematic error at densities corresponding to the joining points between segments of the fit. This motivated implementation of a smoother fit that allows for curvature, specifically a spectral expansion of the adiabatic index (the slope of the EOS in pressure-density space) onto a set of basis functions proportional to the pressure.

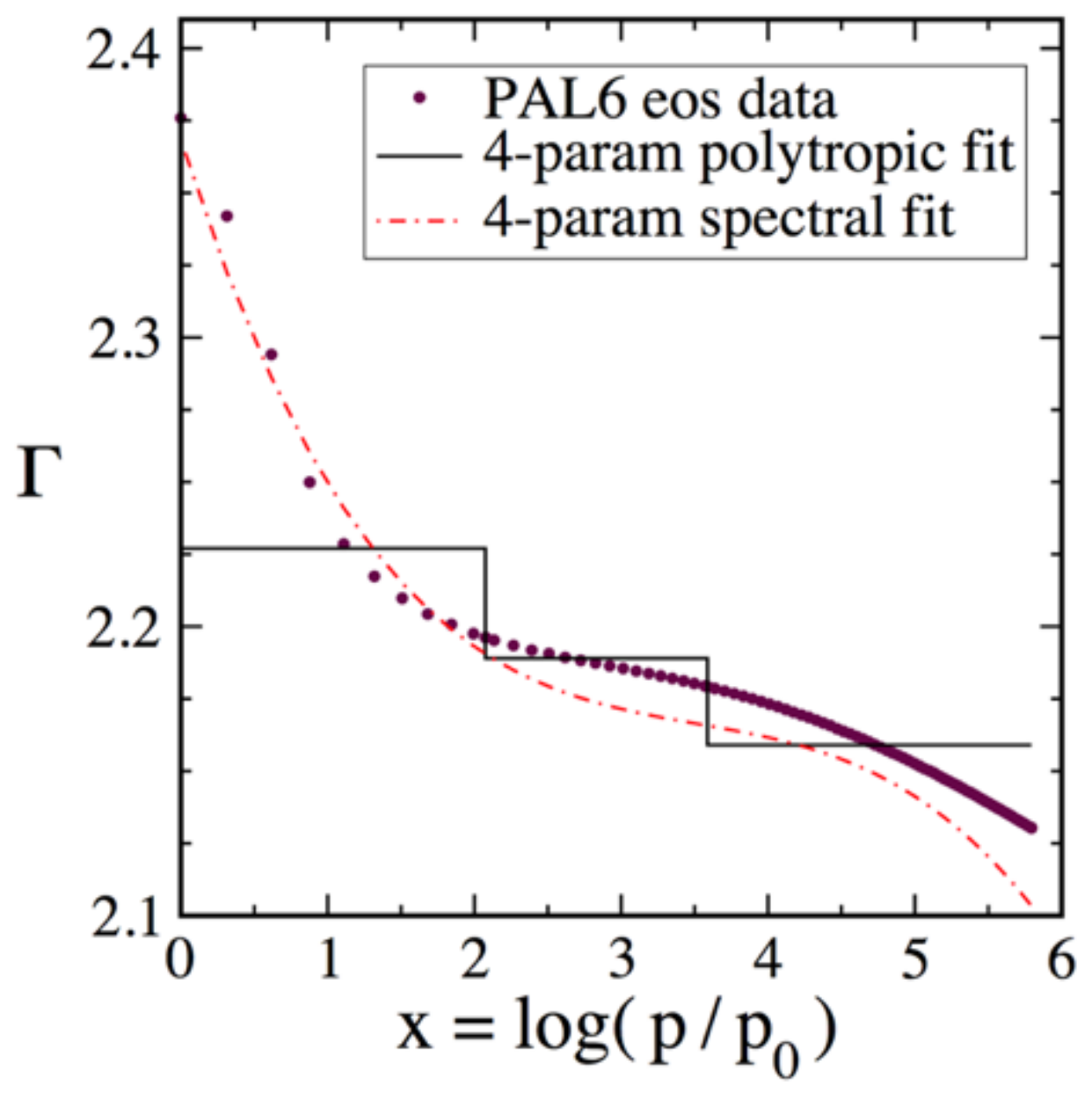

Fig 3: Comparison between polytropic and spectral fits to a candidate EOS [3].

It was found that the spectral expansion is a better fit to candidate EOS’s with fewer parameters. Implementing this new EOS representation into LIGO’s software package was the second half of my summer science project this year.

Preliminary Conclusions

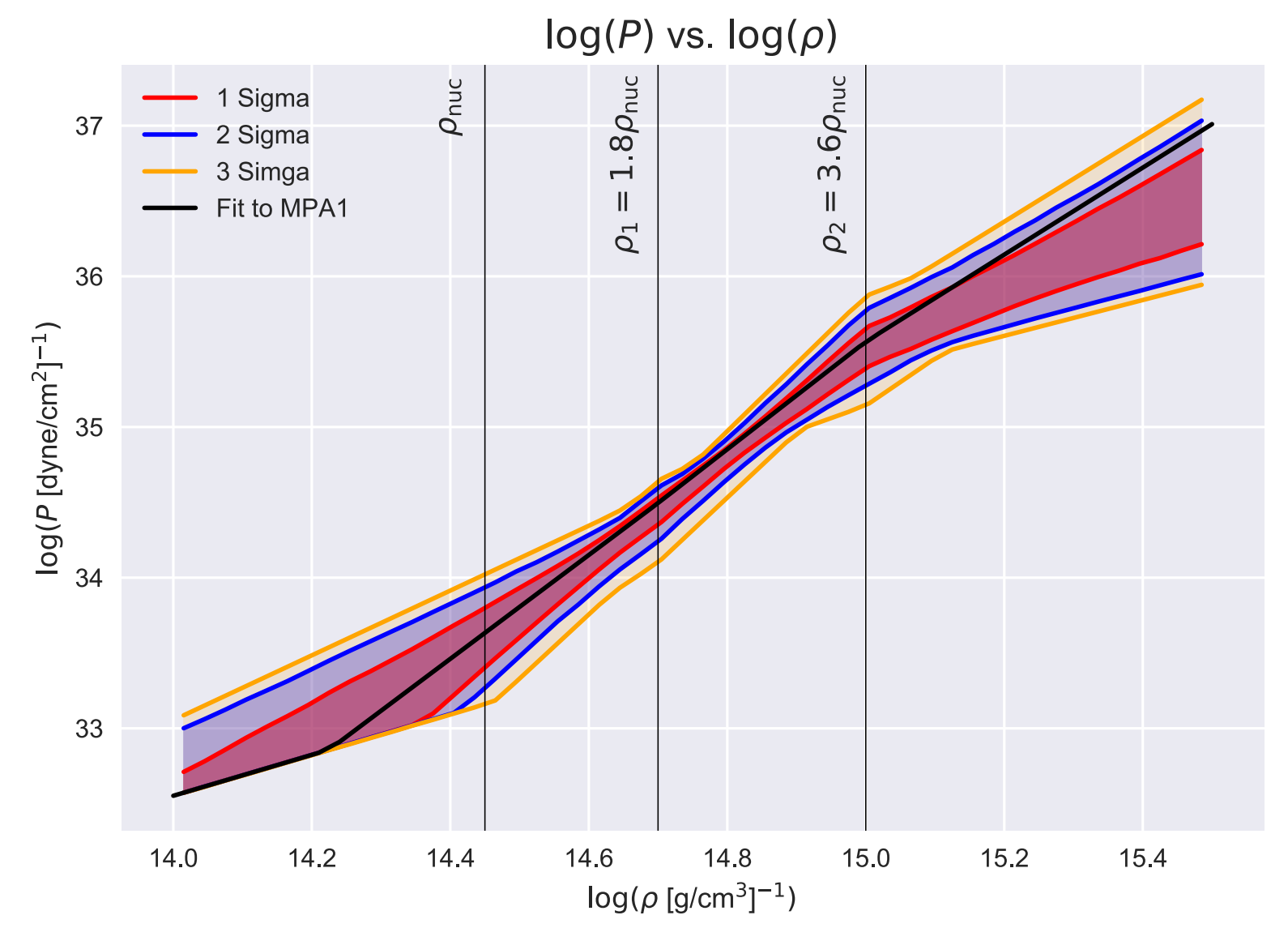

Fig 4. Constraints on EOS for simulated BNS event. Plot courtesy of Burke Irwin.

With Burke Irwin’s data visualization techniques, we were able to translate constraints on individual polytropic fit parameters to overall EOS constraints for a simulated neutron-star GW signal. Consistent with the fit parameter posteriors, the EOS is best constrained at the point p_1, while the constraint on the slope parameters were indistinguishable from the prior.

Future Work

Professor Wade, Burke Irwin, and I will be continuing the work outlined here as an Honors project over the course of the school year. Once the spectral representation of the EOS is implemented into LIGO’s algorithm library, we will perform a comparative study on the effectiveness of each EOS parameterization in constraining the NS EOS from a population of simulated gravitational-wave signals.

References

[1] Wade IV, Leslie, “Gravitational Waves from Rotating Neutron Stars and Compact Binary Systems” (2015). Theses and Dissertations.

[2] Lackey, Benjamin, “Reconstructing the neutron-star equation of state with gravitational-wave detectors from a realistic population of inspiralling binary neutron stars” (2015). Phys. Rev. D 91, 043002.

[3] Lindblom, Lee, “Spectral Representations of Neutron-Star Equations of State” (2010). Phys. Rev. D 82, 103011.

[4] Read, Jocelyn S., Lackey, Benjamin, et al., “Constraints on a phenomenologically parameterized neutron-star equation of state” (2009). Phys. Rev. D 79, 124032

Acknowledgments

I’d like to thank Les Wade, Burke Irwin, Maddie Wade, my group members, the Kenyon College Physics Department, and Summer Science for supporting me in this project.