Visualizing Equation of State Constraints given Gravitational-wave Observations of Binary Neutron Star Systems

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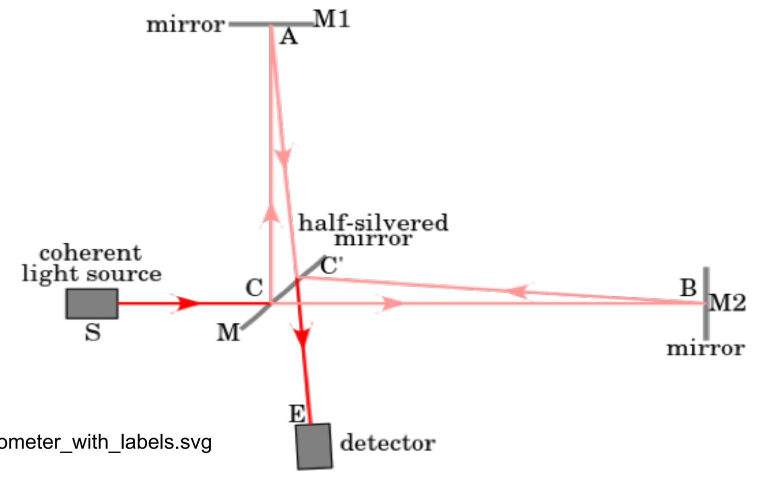
Background

Gravitational Waves are perturbations in spacetime that manifest themselves in distance changes. These phenomena are very difficult to detect due to the fact that contorting spacetime occurs on a very small level. The Laser Interferometer Gravitational Wave Observatory, LIGO, is essentially a Michelson interferometer used to detect gravitational waves. LIGO uses a beam splitter to split a laser down two arms that reflect the light back to a photodiode. When gravitational waves move across the interferometer, they change the length of the arms and create interference patterns at the photodiode. These patterns have information in them about the source parameters of the system from which they originated. One interesting application of this information is constraining the neutron star (NS) equation of state (EOS). An EOS is a relationship between state variables such as pressure and density. There is important EOS information inherent in gravitational waves emitted by binary systems that include NSs.

Figure 1. Michelson Interferometer Schematic showing the path taken by the laser beam. The Michelson Interferometer is the design that LIGO uses for their detectors.

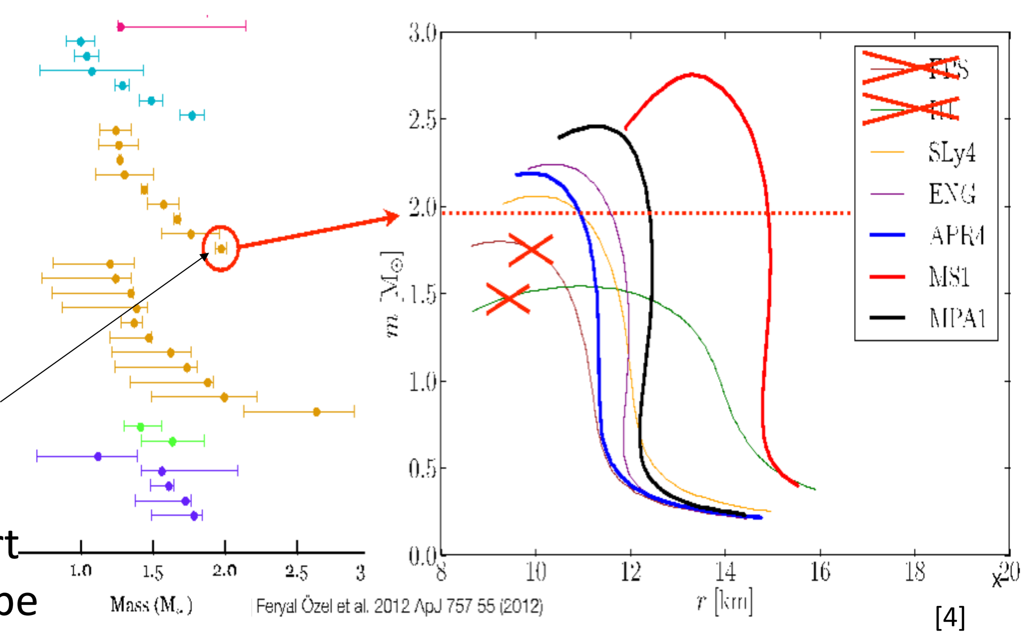


https://en.wikipedia.org/wiki/Michelson_interferometer#/media/File:Michelson_interferometer_with_labels.svg

Neutron Star Equation of State

The neutron star equation of state has been a scientific quandary for many years now. NSs naturally form from the dense neutron matter remaining after the collapse of a large star. With electromagnetic telescopes scientists can accurately constrain the masses but not the radii of these stars. Pressure-density relationships contain the same information as mass-radius relationships, thus using advanced LIGO, we can use the information inherent in a GW signal to potentially make more precise radius measurements. From this, we can establish an EOS relationship between the mass and the radius of a NS.

Figure 2
- Shows 7 candidate NS EOS mass-radius relationships.
- Any mass-radius combination must fall on an EOS
- In 2006, scientists constrained the J1614-2230 NS mass to 2.0 solar masses
- Thus 2 EOSs cannot support a NS this massive and must be thrown out



All of the signals seen by LIGO thus far originated from binary systems. Binary systems evolve from two bodies orbiting each other (the inspiral), to a collision between the two bodies (the merger), and then finally a phase where the mass distributes spherically (the ringdown). Binary systems with a NS will deform near the merger due to extreme tidal forces. These tidal effects are a window to the NS EOS and are detectable using Advanced LIGO. We model the NS EOS as a 4-piece polytrope with the 4 parameters $(\log(p1), \Gamma_1, \Gamma_2, \Gamma_3)$. Constraints on these 4 parameters put constraints on the relationship between pressure and density of nuclear matter at these high densities. In this way, we can measure the NS EOS.

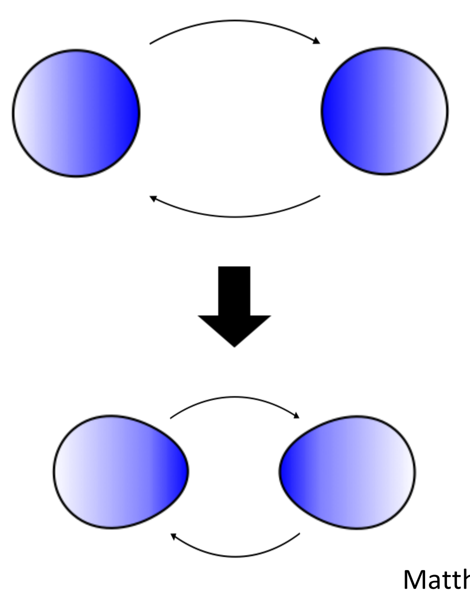


Figure 3. Tidal effect on spherical bodies transform them into "egg" shaped bodies.

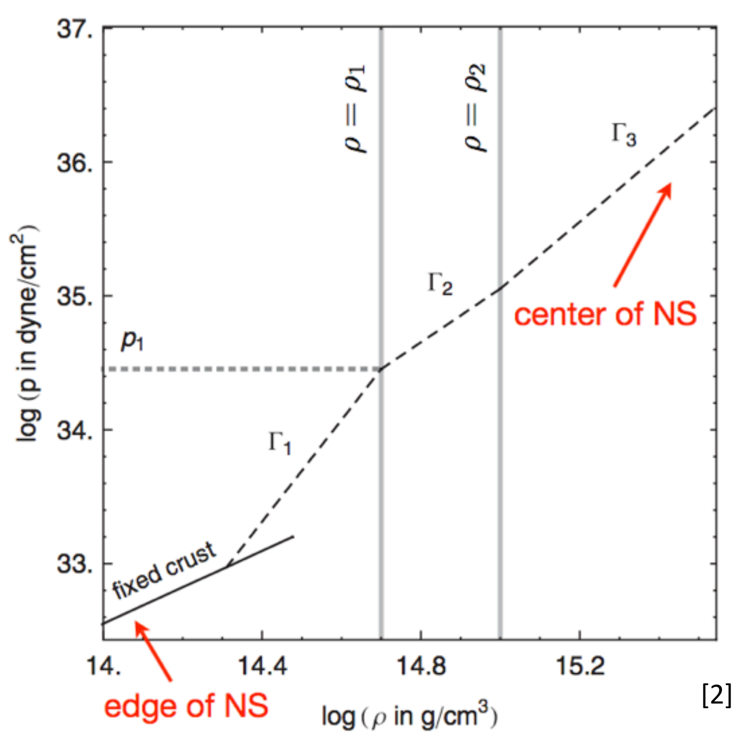


Figure 4. The four piece polytrope model of the NS EOS.

Parameter Estimation

How do LIGO scientists extract the information from a binary NS signal? The answer—parameter estimation. We use Bayes’ Law to estimate the probability of a given signal to have certain parameters given a model.

$$p(\vec{\theta}|d,m) = \frac{\overbrace{p(\vec{\theta}|m)}^{\text{Prior}}\,\overbrace{p(d|\vec{\theta},m)}^{\text{Likelihood}}}{\underbrace{p(d|m)}_{\text{Evidence}}}$$

Posterior

Bayes’ Law mathematically relates the posterior distribution to three main components. The prior is the probability of the parameters given the model. The likelihood is the probability of the data given the parameters and the model. The evidence is the probability of the data given the model. With this law, LIGO scientists use a Markov Chain Monte Carlo (MCMC) to randomly sample parameter space and map out the underlying posterior probability distribution. Figure 5 shows a nice flow chart of how the MCMC works.

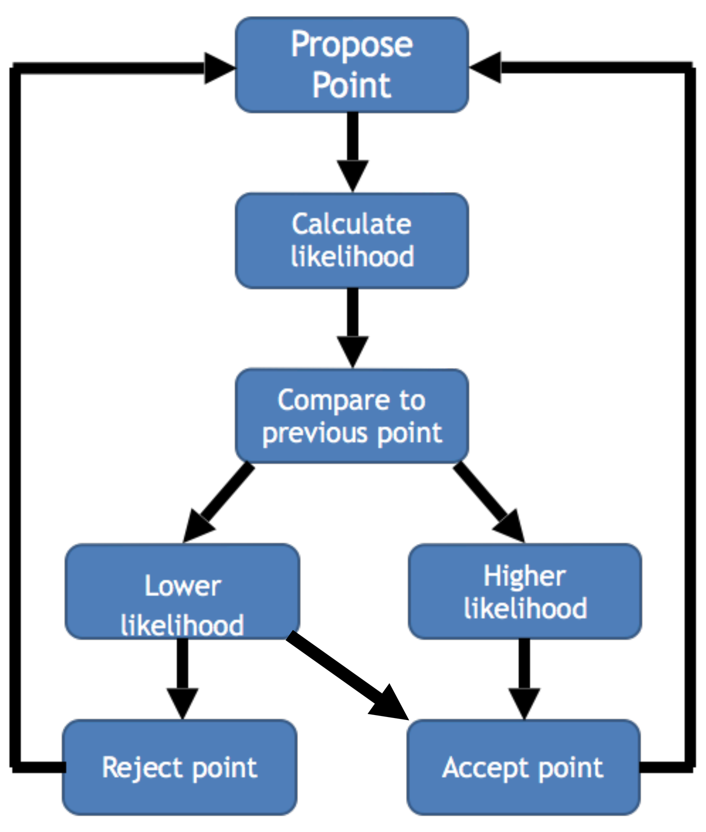


Figure 5. A flowchart of the MCMC process.

Basically, a point in parameter space is proposed and the posterior is computed with those parameters. If the posterior is higher than that of the previous point, the new point is accepted, and if the posterior is lower, the new point is rejected. The first proposed point is always accepted to ensure that the system gets running. There is a random chance that a proposed point with a lower posterior than the current point will be accepted to make sure that the MCMC does not get stuck on a local maximum. The MCMC algorithm results in posterior samples whose density is proportional to the underlying posterior distribution.

1 and 2 Dimensional Plots

My project was to visualize MCMC data I received from my collaborator Matthew F. Carney. My code took the raw posterior samples of the 4 piece polytrope parameters, converted them into a posterior probability distribution, and plotted these distributions in one and two dimensions

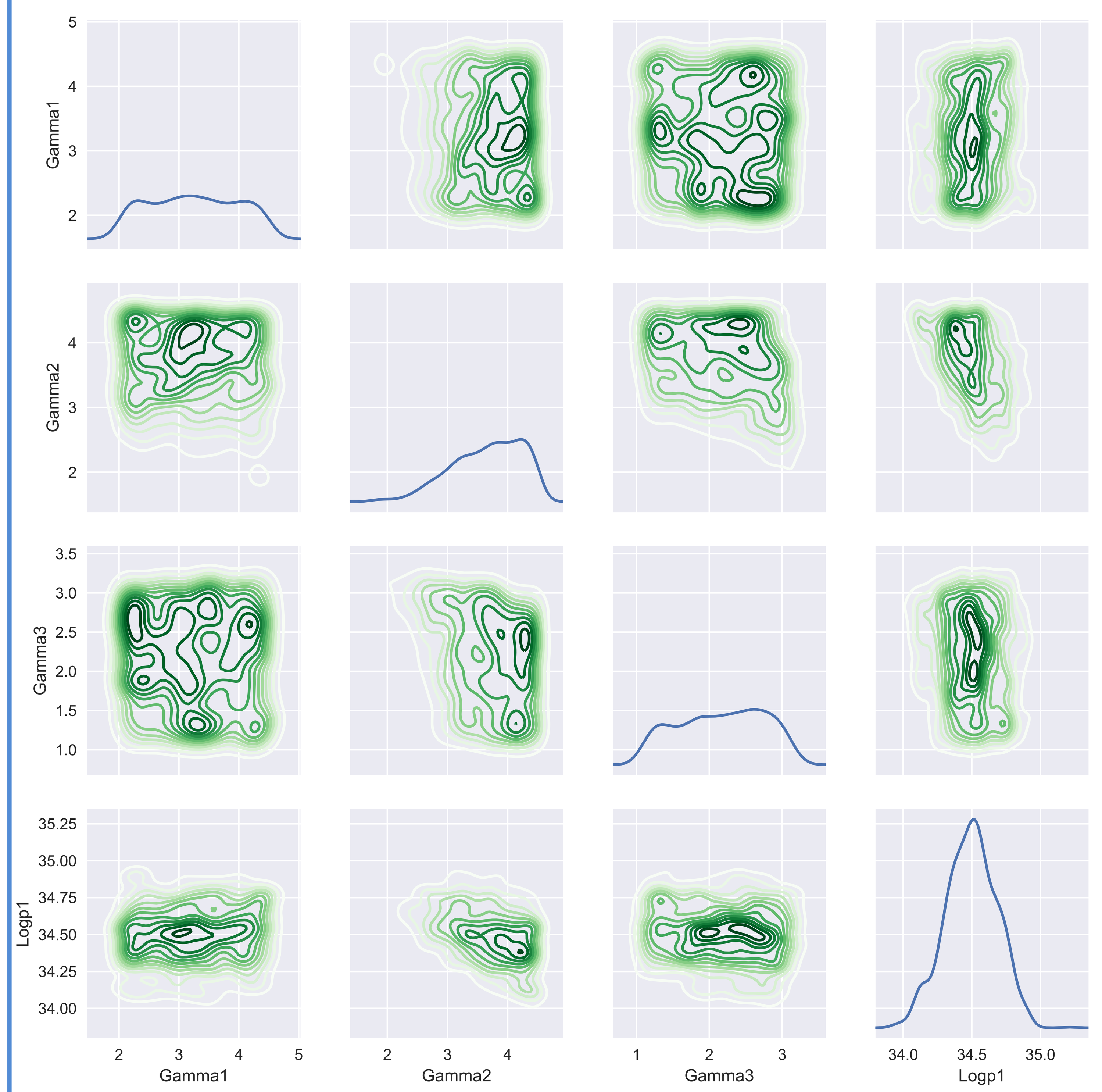


Figure 6. One dimensional visualizations of the 4 piece parameters (diagonal) and two dimensional visualizations of the 4 piece parameters (density plots).

2 Dimensional EOS Plots

The main point of this project was to visualize better constraints on the 4 Piece Polytrope NS EOS model. Through computing the confidence intervals for many values of density, I could constrain the pressure space and thus constrain the entire curve.

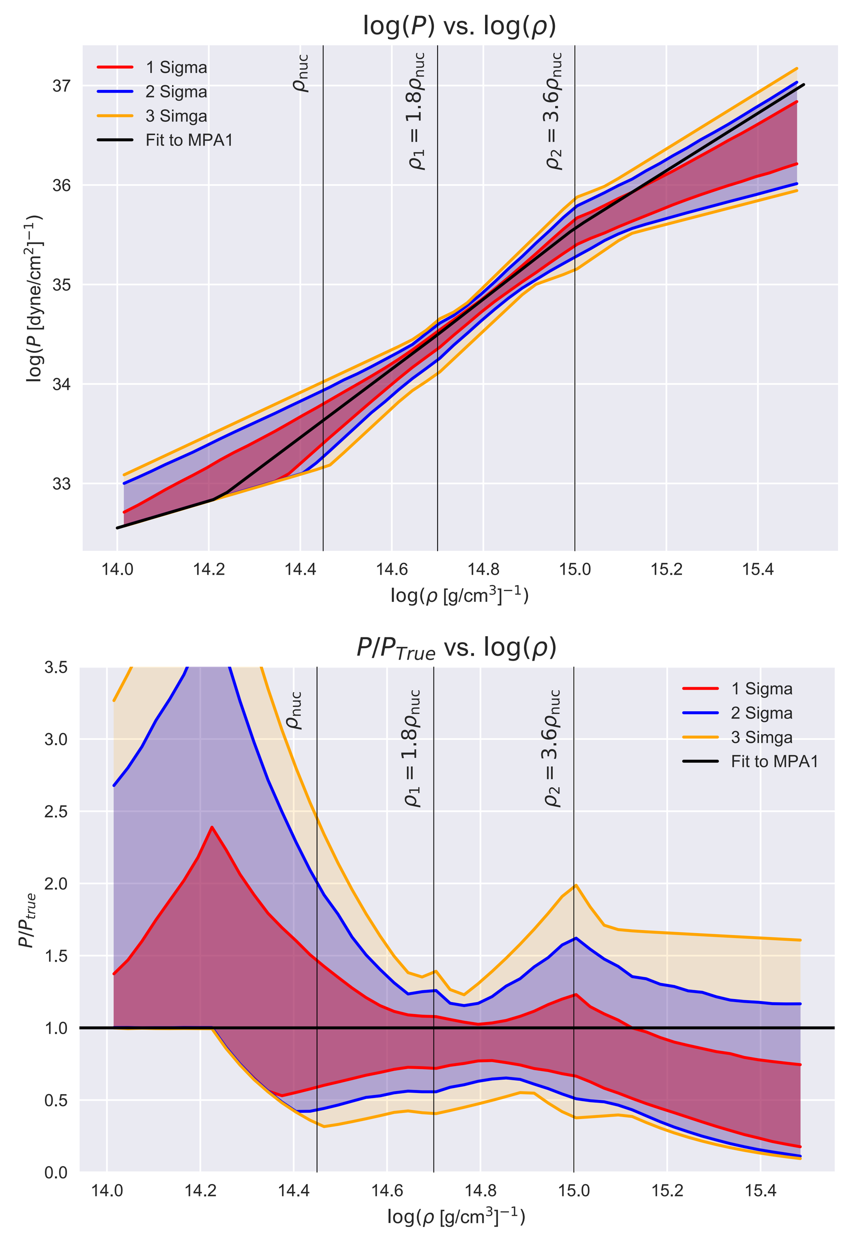


Figure 7. Visualization of MCMC samples in EOS space. This is where we can visually bound the EOS. The top plot is just the 4 Piece polytrope model with 1, 2, and 3 sigma bounds. The bottom plot is the pressure samples with the MPA1 pressure divided out of the samples.

Conclusions and Future Work

The junction points of the 4 piece polytrope model had noticeably larger systematic error, which is most likely just inherent to the type of model we chose. In a physical sense, the relationship between density and pressure is not going to abruptly change when you hit a certain depth of the star. A better fit might be a spectral decomposition of the slope in pressure-density space that will eliminate discontinuities in the fit. This will allow for curvature in the fit and less overall error in the constraining. Additionally, combining multiple MCMC runs can constrain the EOS even better.

Lastly, a large part of my summer went into also creating plots for the mass-radius EOS. These plots are in the final stages of production and should be completed soon.

References

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